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ELEMENTARY PRINCIPLES

OF

ELECTRICITY AND MAGNETISM

THE MACMILLAN COMPANY
NEW YORK BOSTON CHICAGO
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MACMILLAN & CO., LIMITED
LONDON BOMBAY CALCUTTA
MELBOURNE

THE MACMILLAN CO. OF CANADA, LTD.
TORONTO

ELEMENTARY PRINCIPLES

OF

ELECTRICITY AND MAGNETISM

FOR STUDENTS IN ENGINEERING

BY

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New York
THE MACMILLAN COMPANY
LONDON: MACMILLAN & CO., LTD.
1913

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Set up and electrotyped. Published February, 1913.

Norwood Press:
Berwick & Smith Co., Norwood, Mass., U.S.A.

INTRODUCTION.

The object of this text is to develop, in logical order, the more important numerical relations existing among the principal quantities employed in electricity and magnetism. No attempt is made to present any physical theory as to the ultimate nature of electricity and magnetism or any mechanism by which electric and magnetic changes take place. The relations are developed from definitions and elementary laws which are purposely stated in language free from the terms and conventions of any particular physical theory. Only those relations which are fundamental to the design of the various machines and instruments used in engineering practice are developed.

It is intended that this text be used in conjunction with lecture demonstrations involving three types of experiments, viz.:

1. Those presenting elementary phenomena or independent experimental laws upon which the propositions depend.
2. Verification of the principal propositions derived.
3. Illustrations of the more important applications.

Problems are given to illustrate the use of formulæ and to fix in the mind of the student, the conditions to which they apply. A knowledge of mechanics and trigonometry should precede but analytic geometry and calculus may accompany the study of this course.

No attempt is made to furnish the descriptive matter usually presented in such a course. The principal equations are numbered for convenience of reference.

R. H. HOUGH,
WALTER M. BOEHM.

PHILADELPHIA,
June, 1912.

CONTENTS.

CHAPTER I.

MAGNETISM.

The characteristic property of magnetite is its property of exerting a force on small pieces of iron. If by any process a body can be made to acquire this characteristic property of magnetite, it is said to be *magnetized.* The process is called *magnetization,* and the force is called a *magnetic force.* Effects essentially like the characteristic property of magnetite are called *magnetic effects. Magnetism* is defined as the cause of magnetic effects. Assuming the quantity of the cause to be proportional to the quantity of the effect, *quantity of magnetism* of a magnetized body may be defined as that property of the body which is directly proportional to the force it exerts upon a second magnetized body, all other things being kept equal. It is represented by the symbol m.

Poles.—If a magnetized body is suspended free to rotate about a vertical axis, it comes to rest in a definite position. The north-seeking end is called the *north pole* or *positive pole,* and the south-seeking end is called the *south pole* or *negative pole.* Experimentally* it is found that like poles repel each other and unlike poles attract each other; therefore a magnetized body

*Master Peter de Maricourt (Petrus Peregrinus), 1269 A.D.

is said to be *polarized.* The term "pole-strength" is synonymous with the phrase "the quantity of magnetism of a pole."

From this definition it follows that :

1. One pole is equal to a second pole if, under the same conditions, it exerts the same force upon a third pole.

2. One pole is a certain multiple of another pole if, under the same conditions, it exerts that multiple of the force upon a third pole.

3. The force exerted between two poles is directly proportional to the product of their pole-strengths, all other things being kept equal.

$$f \propto m \cdot m'$$

Law of Inverse Squares.—Experimentally it is found that the force exerted between two poles is in the direction of the straight line joining the poles and varies inversely as the square of the distance between them, provided the distance is so great that the poles may be considered as point sources and all other things remain the same.

$$f \propto \frac{1}{r^2}$$

Permeability.—Experimentally it is found that the force exerted between two poles at the same distance, is different for different media. Therefore the force exerted between two poles depends upon some property of the medium in which they are placed. That property of the medium to which the force is inversely proportional, other things being kept equal, is called the *permeability*. It is represented by the symbol μ.

$$f \propto \frac{1}{\mu}.$$

Coulomb's Law for Magnetic Poles.—Collecting these separate statements:

$$f = k \cdot \frac{1}{\mu} \cdot \frac{m \cdot m'}{r^2}$$

In which m = strength of one pole.
m' = strength of the other pole.
r = distance between the poles.
f = force exerted between the poles.
μ = permeability of the medium surrounding the poles.
k = constant of proportionality.

The *unit of pole-strength* is called the electro-magnetic unit of pole-strength if the unit of permeability is that of air; the unit of length, the centimeter; the unit of force, the dyne; and the constant, k, is equal to one. In electro-magnetic units (e.m.u.) Coulomb's law for point sources may be stated as follows:

$$f = \frac{1}{\mu} \cdot \frac{m \cdot m'}{r^2} \tag{1}$$

As an *illustration, unit pole,* in the electro-magnetic system, is a pole of such strength that it exerts a force of one dyne on an equal pole placed at a distance of one centimeter in air.

If m and m' are of the same sign their product is positive and they repel each other; if they are of opposite sign their product is negative and they attract each other.

Magnetic Field.—The region about a magnetic pole, considered with reference to the force exerted by this pole upon another magnetic pole, is called a *magnetic field.* The region about one or more magnetic poles, considered with reference to the forces they exert upon a pole, is called the field due to these poles. *Intensity of the magnetic field,* at a point, is defined as the ratio of the force exerted upon a pole, placed at that point, to its pole strength; provided the pole is so small as to leave the field unaltered at all finite distances. It is represented by the symbol $\mathcal{H}$.

$$\mathcal{H} = \frac{f}{m'} \tag{2}$$

In which m' = pole-strength of the pole placed at the point.

f = force exerted upon the pole.

$\mathcal{H}$ = intensity of the field at that point.

The term "field" is commonly used for "intensity of field." The *unit* of *intensity of field* is called the electro-magnetic unit of intensity, if the unit of force is the dyne and the unit of pole-strength is the electro-magnetic unit of pole-strength.

Solving the equation (2) simultaneously with Coulomb's law (1), the expression for the field due to a single pole, m, at a distance r, is:

$$\mathcal{H} = \frac{1}{\mu} \cdot \frac{m}{r^2} \qquad \text{From (1) and (2).} \tag{3}$$

$\mathcal{H}$ is a vector having the same direction as r. If m is positive the direction of $\mathcal{H}$ is from m; if m is negative the direction of $\mathcal{H}$ is toward m. $\mathcal{H}$ may be resolved into components along X, Y, and Z axes.

$$\mathcal{H}_x = \mathcal{H} \cos \alpha \tag{4}$$

$$\mathcal{H}_y = \mathcal{H} \cos \beta \tag{5}$$

$$\mathcal{H}_z = \mathcal{H} \cos \gamma \tag{6}$$

In which $\cos \alpha$, $\cos \beta$ and $\cos \gamma$ are the direction cosines of $\mathcal{H}$.

$$\mathcal{H} = \sqrt{\mathcal{H}_x^2 + \mathcal{H}_y^2 + \mathcal{H}_z^2} \tag{7}$$

Resultant Field.—The resultant field, at any point, due to n poles is the vector sum of the n separate intensities. If the poles are numbered from 1 to n, the intensity:

$$\mathcal{H}_k = \frac{1}{\mu} \frac{m_k}{r_k^2} \tag{8}$$

represents the intensity of field due to the kth pole. In which r_k is the distance *from* the kth pole to the point at which the intensity is to be determined. As k varies from 1 to n:

$$(9) \qquad \mathcal{H}_x = \frac{1}{\mu} \cdot \sum_{k=1}^{n} \frac{m_k}{r_k^2} \cdot \cos \alpha_k$$

$$(10) \qquad \mathcal{H}_y = \frac{1}{\mu} \cdot \sum_{k=1}^{n} \frac{m_k}{r_k^2} \cdot \cos \beta_k$$

$$(11) \qquad \mathcal{H}_z = \frac{1}{\mu} \cdot \sum_{k=1}^{n} \frac{m_k}{r_k^2} \cdot \cos \gamma_k$$

In which $\mathcal{H}_x$, $\mathcal{H}_y$ and $\mathcal{H}_z$ are the X, Y, and Z components of the resultant intensity $\mathcal{H}_r$.

$$(12) \qquad \mathcal{H}_r = \sqrt{\mathcal{H}_x^2 + \mathcal{H}_y^2 + \mathcal{H}_z^2}$$

$$(13) \qquad \cos \alpha = \frac{\mathcal{H}_x}{\mathcal{H}_r}$$

$$(14) \qquad \cos \beta = \frac{\mathcal{H}_y}{\mathcal{H}_r}$$

$$(15) \qquad \cos \gamma = \frac{\mathcal{H}_z}{\mathcal{H}_r}$$

The resultant intensity due to any number of poles, distributed in any way, can be completely determined in magnitude and direction, at any point with reference to which the poles may be considered as point sources.

If the n poles lie in the same plane and the resultant intensity is to be found at some point in that plane:

$$(16) \qquad \mathcal{H}_r = \sqrt{\mathcal{H}_x^2 + \mathcal{H}_y^2}$$

$$\cos \gamma = 0$$

$$\alpha + \beta = \frac{\pi}{2}$$

(17) $\therefore \qquad \cos\beta = \sin\alpha$

(18) $\qquad \tan\alpha = \dfrac{\mathcal{H}_y}{\mathcal{H}_x} \qquad$ From (13), (14), (17).

If in any region the intensity of the field is everywhere the same in magnitude and direction, the field in that region is said to be *uniform*.

Magnetic Moment.—If a magnetized body is placed in a uniform field, $\mathcal{H}$, the force exerted upon it is:

(19) $$f = \mathcal{H}\Sigma m'$$

Experimentally we find that it may have a motion of rotation or a tendency to adjust itself in a definite direction, but it has no tendency to motion of translation. The resultant force tending to produce translation must therefore be equal to zero. If in a uniform field the force tending to produce translation is equal to zero, the algebraic sum of the pole-strengths must be equal to zero, according to equation (19).

(20) $$\Sigma m' = 0$$

From this it follows that every magnetized body has equal and opposite quantities of magnetism. The body is called a *magnet*. The product of the pole-strength times the distance between the poles is called *magnetic moment*. If the poles may be considered as point sources, the line passing through them is called the *axis of the magnet*. By convention, the direction from the negative to the positive pole is taken as positive.

$$M = m \cdot l \tag{21}$$

In which M = magnetic moment.

m = pole-strength.

l = distance between the poles.

Verification of Coulomb's Law.—Since every proposition in magnetism and electro-magnetism is based directly or indirectly on Coulomb's law for magnetic poles, a rigorous experimental test should be given. Four propositions are necessary for this purpose:

The expression for the moment of force exerted upon a magnet.

The principle of the magnetometer.

The expressions for Gauss' two positions.

Moment of Force.—If a magnet is free to rotate about a line which is perpendicular to a uniform field and to the axis of the magnet, the moment of force producing rotation about this line may be expressed in terms of: the intensity of the field, the magnetic moment of the magnet, and the angle between the axis of the magnet and direction of the field. Let θ = angle between the axis of the magnet and direction of the field. It will always be taken as positive. Let L = resultant moment of force *toward* the position of equilibrium.

Let f_1 = force on the positive pole.

f_2 = force on the negative pole.

L_1 = moment of force due to f_1.

L_2 = moment of force due to f_2.

A counter clockwise moment being considered as positive in Fig. 1.

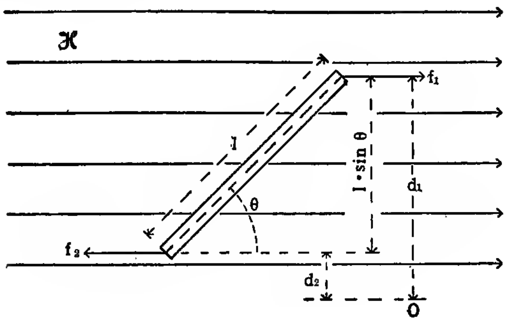


FIG. 1.

$$L = -(L_1 + L_2)$$

$$L_1 = -f_1 \cdot d_1 \qquad \text{By definition.}$$

$$L_2 = -f_2 \cdot d_2 \qquad \text{By definition.}$$

$$L = f_1 \cdot d_1 + f_2 d_2$$

$$f_1 = m\mathcal{H} \qquad \text{From (2).}$$

$$f_2 = -m\mathcal{H} \qquad \text{From (2.)}$$

$$L = m \cdot \mathcal{H} \cdot (d_1 - d_2)$$

$$d_1 - d_2 = l \cdot \sin\theta \qquad \text{Fig. 1.}$$

$$L = m \cdot \mathcal{H} \cdot l \cdot \sin\theta$$

$$M = m \cdot l \qquad (21)$$

(22) $$L = M \cdot \mathcal{H} \cdot \sin\theta$$

In this equation L is defined as the moment of force toward the position of equilibrium and θ is always taken as positive. If L were defined as the moment of force from the position of equilibrium, *i. e.*, positive directions are chosen the same for both L and θ:

$$L = - M \cdot \mathcal{H} \cdot \sin \theta \tag{23}$$

The condition for equilibrium in equation (22) is that the moment must equal zero, or

$$\theta = 0 \tag{24}$$

When a magnetic needle, subject to the above conditions comes to rest, *i. e.*, to a condition of stable equilibrium, its axis lies in the direction of the resultant field.

Magnetometer.—A magnetometer is an instrument for comparing intensities of two magnetic fields at right angles to each other, in terms of the angular deflection of a small magnetic needle. One of the intensities is usually the horizontal component of the earth's field, represented in Fig. 2, by $\mathcal{H}_e$. The second

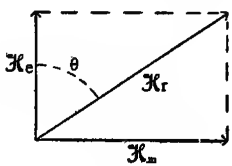


FIG. 2.

field is called the "cross field" and is represented by $\mathcal{H}_m$. $\mathcal{H}_R$ represents the direction and magnitude of the resultant field. θ_m is the angular deflection of the needle.

(25) $$\tan \theta_m = \frac{\mathcal{H}_m}{\mathcal{H}_e}$$ (From Fig. 2.)

Gauss' Positions.—(*a*) Intensity of field at a point in the axis of a magnet.

Let P, Fig. 3, be a point in the axis of a magnet.

$\mathcal{H}_N$ = field at P, due to the north pole.
$\mathcal{H}_S$ = field at P, due to the south pole.
$\mathcal{H}_a$ = resultant field.
d = distance from the middle point of the magnet to the point P.

Let l = distance between the poles.

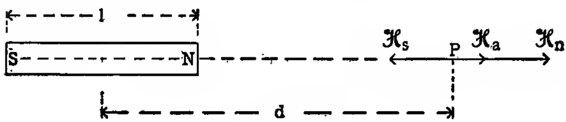


FIG. 3.

$$\mathcal{H}_a = \mathcal{H}_N + \mathcal{H}_S$$

$$\mathcal{H}_N = \frac{m}{\left(d - \frac{l}{2}\right)^2} \qquad (3)$$

$$\mathcal{H}_S = \frac{-m}{\left(d + \frac{l}{2}\right)^2} \qquad (3)$$

(26) $$\mathcal{H}_a = \frac{2 \cdot m \cdot d \cdot l}{\left(d^2 - \frac{l^2}{4}\right)^2}$$

Dividing numerator and denominator by d:

(27) $$\mathcal{H}_a \doteq \frac{2ml}{d^3}$$

If

$$\frac{l^2}{4d^2} \doteq 0$$

$$M = ml$$

(28) $$\mathcal{H}_a \doteq \frac{2M}{d^3}$$

(b) Intensity of field in the median plane of a magnet. Let P be a point, in the median plane of a magnet, at which the intensity is to be found.

r = distance from either pole to the point P.

$\mathcal{H}_P$ = resultant field at P.

$$\mathcal{H}_N = \mathcal{H}_S$$

Resolve $\mathcal{H}_N$ and $\mathcal{H}_S$ into components parallel to the plane and components perpendicular to the plane, at the point P. If the X axis is chosen parallel to the plane:

$$\mathcal{H}_X = \frac{m}{r^2} \cdot \frac{d}{r} - \frac{m}{r^2}\frac{d}{r}$$

(29) $$\therefore \quad \mathcal{H}_X = 0$$

$$\mathcal{H}_y = \frac{m}{r^2} \cdot \frac{l}{2r} - \frac{-m}{r^2} \cdot \frac{l}{2r}$$

(30) $$\therefore \quad \mathcal{H}_y = \frac{m \cdot l}{r^3}$$

$$r = \sqrt{d^2 + \frac{l^2}{4}}$$

(31) $$\mathcal{H}_v = \frac{ml}{\left(d^2 + \frac{l^2}{4}\right)^{\frac{3}{2}}}$$

(32) $$\therefore \quad \mathcal{H}_P = \frac{ml}{\left(d^2 + \frac{l^2}{4}\right)^{\frac{3}{2}}}$$ From (16), (29), (31).

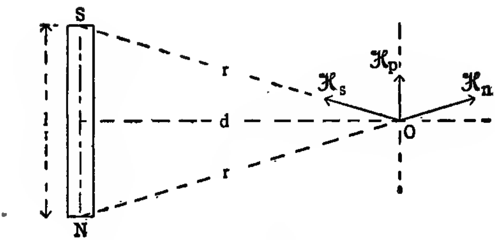


FIG. 4.

(33) $$\mathcal{H}_P \doteq \frac{m \cdot l}{d^3}$$

If $$\frac{l^2}{4d^2} \doteq 0$$

(34) $$\mathcal{H}_P \doteq \frac{M}{d^3}$$ From (21) and (33).

Dividing equation (28) by equation (34):

(35) $$\frac{\mathcal{H}_a}{\mathcal{H}_P} \doteq 2$$

If the magnet is placed successively in these two positions with reference to the needle of a magnetometer, so that the distance, d, is large compared with the length of the magnet, we have, from the principle of the magnetometer:

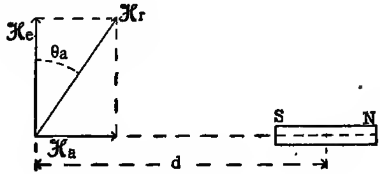


FIG. 5.

(36) $$\frac{\mathcal{H}_a}{\mathcal{H}_e} = \tan \theta_a$$ (25) and Fig. 5.

(37) $$\frac{\mathcal{H}_p}{\mathcal{H}_e} = \tan \theta_p$$ (25) and Fig. 6.

$\therefore$ From (36) and (37), $$\frac{\mathcal{H}_a}{\mathcal{H}_p} = \frac{\tan \theta_a}{\tan \theta_p}$$

Thus by experiment it is found that:

(39) $$\frac{\tan \theta_a}{\tan \theta_p} \doteq 2$$

Verification.—

$$f = \frac{1}{\mu} \cdot \frac{mm'}{r^2}$$ By hypothesis (1).

$$\mathcal{H} = \frac{f}{m'},$$ By definition (2).

$$\mathcal{H} = \frac{1}{\mu} \frac{m}{r^2}$$

$$\frac{\mathcal{H}_a}{\mathcal{H}_p} \doteq 2$$ By deduction (35).

$$\frac{\mathcal{H}_a}{\mathcal{H}_p} \doteq 2 \qquad \text{By experiment (39).}$$

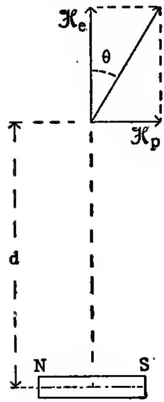


FIG. 6.

In the same way every other deduction made from Coulomb's law can be verified within the limits of experimental error.

$$\therefore \quad f = \frac{1}{\mu}\frac{mm'}{r^2}$$

within the limits of experimental error.

Horizontal Component of the Earth's Field.—The horizontal component of the earth's field is determined by means of the magnetic pendulum and the magnetometer. If a magnet of moment, M, is suspended in the earth's field, free to rotate about a vertical axis, the moment of the force toward the position of equilibrium is expressed by:

$$L = M\mathcal{H}_e \sin\theta \tag{22}$$

The instrument is called a *magnetic pendulum.* If the force toward the position of equilibrium of a body is directly proportional to the distance from that position, the body executes simple harmonic motion. From elementary mechanics we have :

$$T = 2\pi\sqrt{\frac{d}{-a}} \tag{40}$$

In which T = period of the pendulum.
d = distance from the position of equilibrium.
a = linear acceleration from the position of equilibrium.

In the magnetic pendulum, Fig. 7.

θ = angular displacement.
α = angular acceleration from the position of equilibrium. It is always negative.
f_p = force toward the position of equilibrium.

$$f_p = f \cdot \sin\theta \tag{41}$$

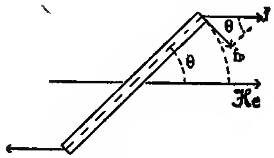


FIG. 7.

For small values of θ the force toward the position of equilibrium is proportional to the distance from the position of equilibrium. ∴ For small angles, the magnetic pendulum executes simple harmonic motion.

$$d = r\theta$$

$$a = r\alpha$$

(42) $$T = 2\pi\sqrt{\frac{\theta}{-\alpha}}$$

The moment of force toward the position of equilibrium is opposed by the moment due to the moment of inertia of the magnet and the angular acceleration, *i. e.*,

(43) $$M\mathcal{H}_e \sin\theta = -I\alpha$$

$$\frac{\sin\theta}{-\alpha} = \frac{I}{M\mathcal{H}_e}$$

If

$$\frac{\sin\theta}{\theta} \doteq 1$$

i. e., θ is small,

(44) $$\frac{\theta}{-\alpha} = \frac{I}{M\mathcal{H}_e}$$

(45) $$T = 2\pi\sqrt{\frac{I}{M \cdot \mathcal{H}_e}}$$ From (42) and (44).

(46) $$M\mathcal{H}_e = \frac{4\pi^2 I}{T^2}$$

In which T is observed and I is calculated from the mass and linear dimensions of the magnet.

If the same magnet, having magnetic moment M, is placed in one of Gauss' positions, at a large distance from the needle of a magnetometer which is located at the place formerly occupied by the magnetic pendulum, we have from the principle of the magnetometer:

$$\frac{\mathcal{H}_p}{\mathcal{H}_e} = \tan \theta_p \tag{37}$$

$$\mathcal{H}_p \doteq \frac{M}{d^3} \tag{34}$$

$$\therefore \quad \frac{M}{\mathcal{H}_e} \doteq d^3 \tan \theta_p . \tag{47}$$

Where θ can be observed and d measured. Divide equation (46) by equation (47) and extract the square root:

$$\mathcal{H}_e = \left(\frac{4\pi^2}{T^2} \cdot \frac{I}{d^3 \tan \theta_p} \right)^{\frac{1}{2}} \tag{48}$$

$\mathcal{H}_e$ can be determined in terms of length, mass and time, and the permeability of air taken as one. Such a determination is called an *absolute determination.*

PROBLEMS.

1. The distance between two poles, in air, is 16 cm. One is + 20 e.m.u.* What must be the pole-strength of the other in order that the force of repulsion be 2 dynes?

Ans. 25.6 e.m.u. of pole strength.

---

* When the expression "a pole of n e.m.u." occurs in a problem it is used instead of the more complicated expression "a pole of n electro-magnetic units of pole-strength." In the same way

2. Four poles are arranged in a straight line at intervals of 10 cm. If the pole-strengths are $+10$, -10, $+30$, and -30 e.m.u. respectively, what are the forces exerted by the first two on the second two, if the medium be air?

Ans. 2.25 dynes attraction on $+30$.
0.42 — dyne repulsion on -30.

3. The force between two magnetic poles is 20 dynes. If each pole be made 4 times its original strength, if the distance between them be made 5 times as great and if the medium be changed so that μ be 10 times as great as before, what force does one exert on the other? Ans. 1.28 dynes.

4. A pole of $+20$ e.m.u. is 60 cm. from one of -80 e.m.u. Find a point in the line passing through the poles, where the intensity of field is zero.

Ans. 60 cm. from $+20$ away from -80.

5. Positive poles of 10, 20, 30, 40, 50, and 60 e.m.u. are placed in counter clockwise order, at the successive angles of a regular hexagon which is inscribed in a circle of radius 10 cm. Find the direction and intensity of field at the center.

Ans. If the line joining the charge $+10$ is chosen as X axis,

$$\mathcal{H}_X = 0.30 \text{ dyne per unit pole.}$$
$$\mathcal{H}_Y = (0.30)\sqrt{3} \text{ dynes per unit pole.}$$
$$\mathcal{H}_r = 0.60 \text{ dyne per unit pole.}$$
$$\alpha = 60°.$$

6. A magnet 15 cm. long and having a pole-strength of 20 e.m.u. is placed at an angle of 30° to the direction of a field the intensity of which is 0.20 dyne per unit pole. Find the moment of the couple acting on the magnet.

Ans. 30 centimeter-dynes.

"a field of n e.m.u." is used instead of "a field of n electromagnetic units of intensity of field," etc.

Note: Problems in this text have been gathered from many sources and no attempt can be made to assign credit to their authors.

7. A bar magnet having a length of 20 cm. and pole-strength 4 e.m.u. is free to rotate about an axis perpendicular to the magnet and to the field. If it be placed at an angle of 60° with the positive direction of the field, what moment of force will tend to turn the magnet, if the intensity of field is 0.2 dyne per unit pole? Draw diagram. Ans. $L = 13.9$ centimeter-dynes.

8. Calculate the moment of force exerted on a bar magnet if the axis of the magnet makes an angle of 30° with a field of intensity 0.18 dyne per unit pole. The length of the magnet is 6 cm. and the strength of each pole is 180 e.m.u.

Ans. 97.2 centimeter-dynes.

9. Find a reference on the subject of terrestrial magnetism and write a paper, considering the following topics:

The earth as a magnet;
Horizontal component $\mathcal{H}_e$;
Vertical component $\mathcal{H}_v$;
Resultant field $\mathcal{H}_R = \sqrt{\mathcal{H}_e^2 + \mathcal{H}_v^2}$;
Dip $\theta = \tan^{-1}\frac{\mathcal{H}_v}{\mathcal{H}_e}$;
Declination;
Diurnal, annual, and secular variations;
Magnetic chart.

10. Given a magnet of magnetic moment 120 e.m.u. and 10 cm. long. Find the intensity of field 16 cm. from the middle, along the axis of the magnet. Ans. .0719 dyne per unit pole.

11. Two magnets 10 cm. long, of equal pole-strengths, are placed with their axes in the same straight line and opposite poles facing each other. If the distance between their centers is 30 cm. what is the intensity of field midway between the two?

Ans. $3m/200$ dynes per unit pole.

12. Calculate the intensity of field at a point 40 cm. from the center, along the axis of a magnet, 6 cm. long and of pole-strength 160 e.m.u. Find the rotational moment exerted on a very short magnet of moment 20 e.m.u. placed at this point with its axis inclined at 30° to that of the larger magnet.

Ans. .0304 dyne per unit pole.
0.304 centimeter-dyne.

13. A magnet 20 cm. long and of pole-strength 9 e.m.u. forms one side of an equilateral triangle. Find the direction and intensity of field at the opposite vertex.

(2) A magnet of pole-strength 2 e.m.u. and 1 cm. long is placed with its center at the opposite vertex, and its + pole pointing toward the + pole of the larger magnet. Find the moment of force on the small magnetic needle.

Ans. .0225 dyne per unit pole.
.0389 centimeter-dyne.

14. Find the intensity of field at a point 14 cm. from the center in the median plane of a magnet which is 6 cm. long and of pole-strength 160 e.m.u. Calculate the force on a pole of + 80 e.m.u. if placed at this point.

Ans. 0.326 dyne per unit pole.
26.1 − dynes.

15. What error is involved in using the approximate formula to determine the intensity of field due to a magnet 20 cm. long, at a point on the median plane, 80 cm. from a magnet?

Ans. The approximate formula gives a result 2.3 per cent. larger than it should be.

16. A magnetometer needle is placed, free to rotate about a vertical axis, in the earth's field of horizontal intensity 0.2 dyne per unit pole. In the same magnetic meridian and in the same horizontal plane a magnet of moment 1600 e.m.u. is placed at right angles to the field, at a distance 80 cm. from the magnetometer needle. Find the deflection of the needle.

Ans. $\log \tan \theta = 8.194 - 10.$
$\theta = 53.7'.$

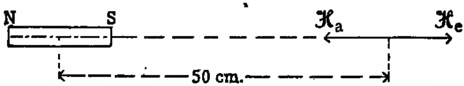
17. By hanging a magnet of moment M in the earth's field $\mathcal{H}_e$, and observing the period of oscillation, the value of $M\mathcal{H}_e$ is calculated to be 70 e.m.u. Find the strength of the earth's field if this magnet (length 5 cm.) deflects a compass needle 1° 57′ from the magnetic meridian when it is placed 50 cm. magnetically east of the needle (tan 1° 57′ = .035).

Ans. 0.178 + dyne per unit pole.

18. A magnet of moment 225 e.m.u. and moment of inertia 102 g.cm^2. is suspended in the earth's field $\mathcal{H}_e$, and is found to execute 25 complete vibrations in 4 min. and 3 sec. What is the horizontal intensity of the earth's field?

Ans. 0.19 – dyne per unit pole.

19. A short bar magnet of large magnetic moment is placed with its axis horizontal but pointing southward. The horizontal component of the earth's field is 0.16 dyne per unit pole. At a comparatively large distance, 50 cm., horizontally north of the magnet the horizontal component of the resultant field is found to be equal to zero. What is the horizontal component of the resultant field at a distance of 100 cm. east from the magnet?



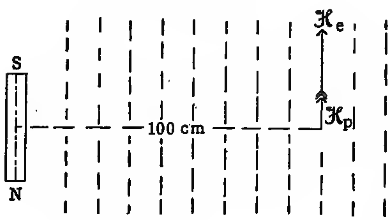
Let $\mathcal{H}_e$ = horizontal component of the earth's field.
$\mathcal{H}_a$ = intensity of field due to the magnet.
d = distance from the magnet to the point.
M = magnetic moments of the magnet.

$$\mathcal{H}_e - \mathcal{H}_a = 0$$

$$\mathcal{H}_a = \frac{2M}{d^3}$$

$$0.16 = \frac{2M}{(50)^3}$$

$$\therefore \quad M = (0.5)(0.16)(50)^3$$



$$\mathcal{H} = \mathcal{H}_e + \mathcal{H}_p \qquad \text{From figure.}$$

$$\mathcal{H}_p = \frac{M}{d^3} \tag{34}$$

$$\mathcal{H}_p = \frac{(0.08)(50)^3}{(100)^3}$$

$$\therefore \quad \mathcal{H} = 0.16 + 0.01$$

$\therefore \quad \mathcal{H} = 0.17$ dyne per unit pole.

CHAPTER II.

MAGNETIC INDUCTION.*

The product of the permeability times the intensity, of a magnetic field, at a point is called the *magnetic induction* at that point. It is represented by the symbol β.

(50) $$\beta = \mu \cdot \mathcal{H}$$

In which $\mathcal{H}$ = intensity of field at the point in the medium.

μ = permeability of the medium.

β = magnetic induction at the point.

The induction is a vector quantity having the same direction as the intensity and differing from it in magnitude if the permeability is not that of air.

The induction due to a single pole, m, is:

(51) $$\beta = \frac{m}{r^2} \qquad \text{From (3) and (50).}$$

β may also be resolved into components along X, Y and Z axes. See equations (4) to (18).

(52) $$\beta_x = \mu \cdot \mathcal{H}_x$$

(53) $$\beta_x = \sum_{k=1}^{n} \frac{m_k}{r_k^2} \cdot \cos \alpha_k \quad \text{From (52) and (9).}$$

* Students who are interested in theories of electricity and magnetism will find excellent discussions in College Physics, by J. O. Reed and K. E. Guthe. The Macmillan Co.

Similar expressions give the values for the Y and the Z components. The induction due to a number of magnetic poles is independent of the permeability and depends only upon the magnitude and distribution of the poles.

Continuity of Induction.—Let *abc*, Fig. 8, be a surface separating two media.

Let m_1, m_2, m_3, m_4 be poles in the media.
Let μ_1 = permeability of medium I.
μ_2 = permeability of medium II.
Let P_1 and P_2 be points infinitesimally near one another, but in the media I and II respectively.
Let β_1 and β_2 be the induction at the points P_1 and P_2 respectively.
Let $\mathcal{H}_1$ and $\mathcal{H}_2$ be the intensities of field at P_1 and P_2 respectively.
k = the subscript number of the pole.
r_{k1} = the distance *from* the pole k *to* the point P_1.
α_{k1} = the angle between $\mathcal{H}_1$ and r_{k1}.

$$\mathcal{H}_1 = \frac{1}{\mu_1}\sum_{k=1}^{n}\frac{m_{k1}}{r_{k1}^2}\cdot\cos\alpha_{k1} \tag{9}$$

$$\mathcal{H}_2 = \frac{1}{\mu_2}\sum_{k=1}^{n}\frac{m_k}{r_{k2}^2}\cdot\cos\alpha_{k2}$$

If there are no poles between P_1 and P_2, and if P_1 approaches P_2;

$$r_{k1} \doteq r_{k2}$$

Also

$$\alpha_{k1} \doteq \alpha_{k2}$$

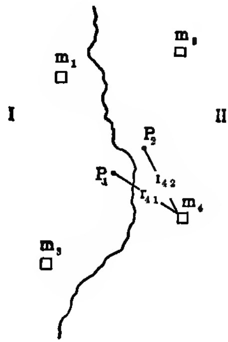


FIG. 8.

(54) $$\mu_1 \mathcal{H}_1 \doteq \mu_2 \mathcal{H}_2$$

(55) $$\therefore \beta_1 \doteq \beta_2 \qquad \text{From (54) and (50).}$$

In going through a surface separating media of different permeabilities, the induction due to all magnetism except that infinitesimally near the point of transit is continuous.

Induction through an element of surface is defined as the product of the normal component of the induction times the area of the element. It is represented by the symbol, φ.

(56) $$\varphi = S \cdot \beta \cdot \cos \alpha$$

In which α = angle between β and the normal to the surface.

S = an element of surface so small that β and α are constant.

The *total normal induction* or total flux through a surface, is defined as the summation of the normal induction over the elements of which the surface is composed. It is represented by the symbol, Φ.

(57) $$\Phi = \Sigma\varphi$$

(58) $$\Phi = \Sigma\beta \cdot S \cdot \cos\alpha$$ From (56) and (57).

Φ is a scalar quantity. It is independent of the permeability and depends upon the quantity and distribution of the magnetism.

Gauss' Theorem.—(*a*) *Poles Inside a Closed Surface.* Let *abc*, Fig. 9, represent the projection of any completely closed surface on a plane.

Let m_1 = a pole at any point within the closed surface.

P = any point on the surface.

Let S be an element of surface, at P, cut out by a solid angle with apex at m_1.

Let r_1 = distance from m_1 to P.

PQ = normal to S.

α_1 = angle between the positive direction of the induction, β, and the positive direction of the normal, PQ. The positive direction of the normal is, by convention, taken outward.

$$\varphi_1 = S \cdot \beta_1 \cdot \cos\alpha_1 \qquad (56)$$

$$\therefore \quad \varphi_1 = S \cdot \frac{m_1}{r_1^2} \cdot \cos\alpha_1$$ From (56) and (51).

Let S' be an element of surface cut out, on a sphere of radius r_1, by the same solid angle as the element S. (See appendix on solid angles.)

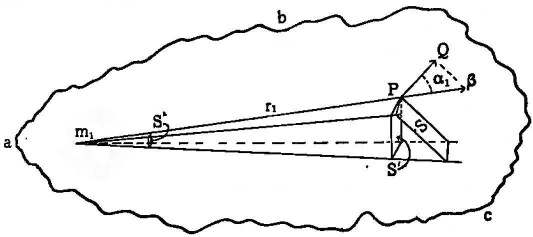


FIG. 9.

$$S' = S \cos \alpha_1 \quad \text{For elements of surface.}$$

$$\therefore \quad \varphi_1 = \frac{S'}{r_1^2} \cdot m_1$$

Let S'' be an element of surface, cut out on a unit sphere with center m_1, by the same solid angle as S and S'.

$$S'' = \frac{S'}{r_1^2}$$

$$\therefore \quad \varphi_1 = m_1 S''$$

$$\Phi_1 = \Sigma \varphi_1 \qquad (57)$$

$$\therefore \quad \Phi_1 = m_1 \Sigma S''$$

For a unit sphere

$$\Sigma S'' = 4\pi$$

(59) $$\therefore \quad \Phi_1 = 4\pi m_1$$

The total normal induction, over a closed surface, due to a single pole m, is equal to 4π times the pole-strength.

Let

$$\Phi = \Phi_1 + \Phi_2 + \Phi_3 + \cdots \Phi_n$$

In which

$$\Phi_1 = 4\pi m_1$$

$$\Phi_2 = 4\pi m_2$$

etc.

$$\therefore \quad \Phi = 4\pi(m_1 + m_2 + m_3 + \cdots m_n)$$

$$\Phi = 4\pi m \tag{60}$$

In which $m =$ the algebraic sum of the poles enclosed by the surface.

$\Phi =$ the total normal induction over the surface.

The total normal induction over any closed surface, due to any number of poles inside, is equal to 4π times the algebraic sum of the quantities of magnetism enclosed.

(*b*) *Poles Outside the Closed Surface.*—Let m_1 be any pole at a point outside the closed surface. A solid angle will intercept the closed surface at S_1 and S_2 (Fig. 10) or some even number of elements of surface. For each solid angle there will therefore be two elements of surface, S_1 and S_2.

Let φ' be the induction through these two elements.

$$\varphi' = S_1 \cdot \beta_1 \cos \alpha_1 + S_2 \cdot \beta_2 \cdot \cos \alpha_2 \tag{56}$$

Proceeding as before,

$$S_1 \cdot \beta_1 \cdot \cos \alpha_1 = - m_1 S''$$

$$S_2 \cdot \beta_2 \cdot \cos \alpha_2 = m_1 S''$$

(61) $$\therefore \quad \varphi' = 0$$

$$\therefore \quad \Sigma\varphi' = 0$$

(62) $$\therefore \quad \Phi' = 0$$

The total normal induction due to any pole on the outside is equal to zero. ∴ The total normal induction over any closed surface is equal to 4π times the

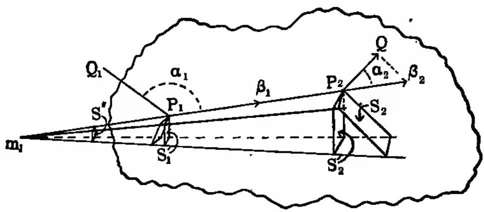


FIG. 10.

algebraic sum of the poles on the inside and is independent of the magnitude and distribution of the poles on the outside.

Application of Gauss' Theorem.—In the application of Gauss' theorem to the demonstration of other propositions an imaginary closed surface is chosen in such a manner that part of the surface is parallel to the direction of the induction and part is perpendicular to it. The total normal induction will be.

(63) $$\Phi = \Phi_1 + \Phi_2 \qquad \text{From (57).}$$

In which Φ_1 is the normal induction over that part of the surface which is parallel to the direction of the induction.

(64) $$\Phi_1 = 0 \qquad \text{From (58).}$$

In (63), Φ_2 is the normal induction over that part of the surface, which is normal to the induction.

(65) $$\Phi_2 = \Sigma\beta S \qquad \text{From (58).}$$

(66) $$\therefore \quad \Phi = \Sigma\beta S \quad \text{From (63), (64), (65)}$$

If the induction is constant over that part of the surface to which it is normal, we may let:

$$\Sigma S = S$$

In which S is the entire surface, normal to the induction.

(67) $$\Phi = \beta \cdot S$$

Variation of Induction through an Element of a Magnetized Surface.—Let *abc*, Fig. 11, represent a surface between two media, I and II, having permeabilities μ_1 and μ_2. Let P_1 and P_2 be two points in the media, on opposite sides of the surface and infinitesimally near it.

Imagine an elementary closed surface lying partly in medium I and partly in medium II. Let S be the element of the surface *abc*, lying within the imaginary closed surface. Let β_1'' and β_2'' be the inductions at P_1 and P_2 respectively, due to the magnetism on the element, S. Imagine the elementary closed surface of such a form that part of it lies directly in the direction of the induction and part of it is perpendicular to the induction at P_1 and P_2.

Let m = quantity of magnetism on the element, S.
$\mathfrak{J}$ = surface density of the magnetism on S, *i. e.*, the pole-strength per unit area, or intensity of magnetization.

(68) $$\mathfrak{J} = \frac{m}{S}$$

If m is positive, $\mathfrak{J}$ is positive and the direction of the induction is away from the surface S. If then the positive direction of the X-axis is taken as the positive direction of the induction,

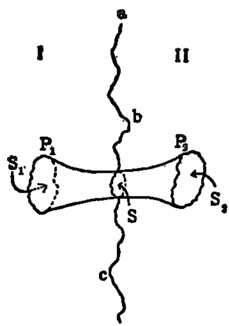


FIG. 11.

(69) $$\Phi = -\beta_1''S_1 + \beta_2''S_2 \qquad \text{From (66).}$$

In which S_1 and S_2 are those parts of the closed surface which are normal to β_1'' and β_2'' respectively.

$$\Phi = 4\cdot\pi\cdot m \qquad \text{From (60).}$$

$$m = \mathfrak{J}S \qquad \text{From (68).}$$

$$4\cdot\pi\cdot\mathfrak{J}\cdot S = -\beta_1''S_1 + \beta_2''S_2$$

As P_1 approaches P_2, $S_1 \doteq S$ and $S_2 \doteq S$

(70) $$\beta_2'' \doteq \beta_1'' + 4\cdot\pi\cdot\mathfrak{J}$$

The induction, due to the magnetism on the element of surface, in going through the element, changes by 4π times the surface density of the magnetism on the element. The induction due to all other magnetism, whether on the same surface, *abc*, or on other magnets, is continuous through the surface. The normal components will therefore be continuous.

$$\beta_1' = \beta_2' \qquad (55)$$

For normal components:

$$\beta_1' \cos \alpha_1 = \beta_2' \cos \alpha_1$$

In which α_1 is the angle between the induction and the normal to the surface. Add and let:

$$\beta_2' \cos \alpha_1 + \beta_2'' = \beta_2 \cos \alpha$$

$$\beta_1' \cos \alpha_1 + \beta_1'' = \beta_1 \cos \alpha$$

(71) $$\beta_2 \cos \alpha = \beta_1 \cos \alpha + 4\cdot\pi\cdot\mathfrak{J}$$

In which α is the angle between the resultant induction and the normal to the surface. If the resultant induction is normal to the surface:

$$\cos \alpha = 1$$

(72) $$\beta_2 = \beta_1 + 4 \cdot \pi \cdot \mathfrak{J}$$

PROBLEMS.

20. What is the total flux from a pole of 150 e.m.u.?

Ans. $600\ \pi$ e.m.u. of flux.

21. What is the total flux from a unit pole?

Ans. $4\ \pi$ e.m.u. of flux.

22. A magnetic pole of 50 e.m.u. is immersed in an atmosphere of permeability 0.99 times that of air. A sphere of 10 cm. radius has its center at the pole. Find the total flux through the surface of the sphere. Also find the induction and the intensity of field at the surface of the sphere.

Ans. Total flux = $200\ \pi$ e.m.u. of flux.
Intensity = 0.505 dyne per unit pole.
Induction = 0.5 e.m.u. of induction.

23. Magnetic poles of +10, −10, +30, and −30 are placed in succession at the corners of a square 10 cm. on each side. A sphere of radius 11 cm. is placed with its center at the pole +10. What is the total normal induction over the sphere; due to all the poles; due to each pole if taken alone?

Ans. For all, $-120\ \pi$ e.m.u. of flux.
For +10, $40\ \pi$ e.m.u. of flux.
For −10, $-40\ \pi$ e.m.u. of flux.
For +30, 0 e.m.u. of flux.
For −30, $-120\ \pi$ e.m.u. of flux.

24. A bar magnet is uniformly magnetized at its ends. If its cross-section is a rectangle of 0.5 cm. × 2.5 cm., and the poles are of 625 e.m.u. of pole-strength, what is the intensity of magnetization? Ans. 500 e.m.u. of pole-strength per sq. cm.

25. A bar magnet 1 sq. cm. in cross-section and 40 cm. long is uniformly magnetized at its ends to 200 e.m.u. of pole-strength per sq. cm. of cross-section. What is the induction at one pole due to the other pole? What is the difference between the induction just outside one pole and just "inside" the same pole?

Ans. 0.125 e.m.u. of induction.
800π e.m.u. of induction.

26. A bar magnet 50 cm. long has a square cross-section of ½ cm. side. It is uniformly magnetized at its ends to pole-strengths of 100 e.m.u. What is the magnetic induction along the axis at a distance of 25 cm. from the end? What is the induction at the middle point of the magnet?

Ans. 0.143 e.m.u. of induction.
0.320 e.m.u. of induction

27. Two bar magnets 10 cm. long, of comparatively small cross-section, and pole strengths of 5 and 6 e.m.u. respectively, are placed with their axes parallel. The distance between their centers measured perpendicular to their axes is 20 cm., and their positive poles point in opposite directions. What is the magnetic induction at a point half way between the positive poles?

The induction at O due to -50 is from O to D. Its magnitude is:

$$\beta_1 = \frac{50}{\overline{OD}^2} \tag{51}$$

The induction at O due to $+50$ is from O to B. Its magnitude is:

$$\beta_2 = \frac{50}{\overline{OC}^2}$$

The induction at O due to $+60$ is from O to C. Its magnitude is:

$$\beta_3 = \frac{60}{\overline{OB}^2}$$

The induction at O due to -60 is from O to A. Its magnitude is:

$$\beta_4 = \frac{60}{\overline{OA}^2}$$

From the figure:

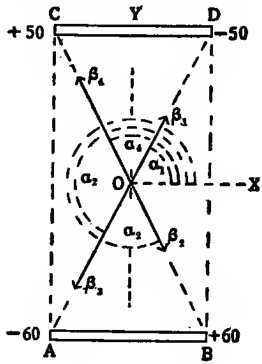
$$\overline{OD} = \overline{OC} = \overline{OB} = \overline{OA}$$

$$\overline{OB}^2 = 5^2 + 10^2 \qquad \text{From the right triangle.}$$

$$\overline{OB} = (125)^{1/2}$$

Taking X components:

$$\beta_X = \beta_1 \cos \alpha_1 + \beta_2 \cos \alpha_2 + \beta_3 \cos \alpha_3 + \beta_4 \cos \alpha_4 \qquad (53)$$



$$\cos \alpha_2 = + \cos \alpha_1$$ From the figure.
$$\cos \alpha_3 = - \cos \alpha_1$$
$$\cos \alpha_4 = - \cos \alpha_1$$

$$\therefore \quad \beta_X = \frac{50}{125} \cos \alpha_1 - \frac{60}{125} \cos \alpha_1 - \frac{60}{125} \cos \alpha_1 + \frac{50}{125} \cos \alpha_1$$

$$\cos \alpha_1 = \frac{5}{(125)^{1/2}}$$

$$\therefore \quad \beta_X = 2 \cdot \frac{50}{125} \cdot \frac{5}{(125)^{1/2}} - 2 \cdot \frac{60}{125} \cdot \frac{5}{(125)^{1/2}}$$

$\therefore \quad \beta_X = 0.357 - 0.429$

$\therefore \quad \beta_X = - 0.072$ e.m.u. of induction.

Taking Y components:

$$\beta_Y = \beta_1 \sin \alpha_1 + \beta_2 \sin \alpha_2 + \beta_3 \sin \alpha_3 + \beta_4 \sin \alpha_4$$

$$\sin \alpha_2 = - \sin \alpha_1$$ From the figure.
$$\sin \alpha_3 = - \sin \alpha_1$$
$$\sin \alpha_4 = + \sin \alpha_1$$

$$\beta_Y = \frac{50}{125} \cdot \sin\alpha_1 + \frac{60}{125} \cdot \sin\alpha_1 - \frac{60}{125} \cdot \sin\alpha_1 - \frac{50}{125} \cdot \sin\alpha_1$$

$\beta_Y = 0$

$\beta_R = (\beta_X^2 + \beta_Y^2)^{1/2}$ Similar to (16).

$\therefore\ \therefore$ $\beta_R = .072$ e.m.u. of induction irrespective of the medium which surrounds the magnets.

$\dfrac{\beta_Y}{\beta_X} = \tan\alpha$ Similar to (18)

$\therefore\ \therefore$ $\tan\alpha = \dfrac{0}{-.072}$

$\therefore\ \therefore$ $\alpha = 180°$

CHAPTER III.

ELECTROSTATICS.

The characteristic property of amber or electron is its property of exerting a force on small particles when it is rubbed. If by any process a body is made to acquire this characteristic property of electron, it is said to be *electrified* or *charged.* The process is called *electrification* and the force is called an *electrostatic force.* Effects essentially like the characteristic property of electron are called *electrical effects. Electricity* is defined as the cause of electrical effects. Assuming the quantity of the cause to be proportional to the quantity of the effect, the *quantity of electricity* of an electrified body may be defined as that property of the body which is directly proportional to the force it exerts on a second electrified body, all other things being kept equal. It is represented by the symbol q. The term "charge" is synonymous with the term "quantity of electricity."

Electroscope.—An electroscope is a device for showing the presence of an electric charge. A simple form of electroscope consists of a pith-ball covered with gold-leaf and suspended by a silk fiber. A body which indicates no charge is said to be *uncharged.* A glass rod, when rubbed with silk, is found, by the electroscope, to be charged. It repels a second glass rod charged by the same method. A rod of hard rubber, if rubbed with cat's-fur, is found, by the electroscope, to be charged. It repels a second rod of hard rubber charged by the same method, but attracts a glass rod charged as above. Therefore there are two kinds of charges. The charge on the hard rubber, by convention, is called a *negative charge* and the charge on the glass is called *positive charge.*

Du Fay's Law.—About the year 1734, du Fay discovered that there were two kinds of electrification. He also formulated the law that like charges repel each other and unlike charges attract each other.

From the definition of quantity of electricity or quantity of charge, it follows that:

1. One charge is equal to a second charge if, under the same conditions, it exerts the same force upon a third charge.

2. One charge is a certain multiple of another charge if, under the same conditions, it exerts that multiple of the force upon a third charge.

3. The force exerted between two charges is directly proportional to the product of the charges, all other things being kept equal.

$$f \propto q \cdot q'$$

Law of Inverse Squares.—Experimentally it is found that the force exerted between two charges is in the direction of the straight line joining the two charges and varies inversely as the square of the distance between them, provided the distance is so great that the charges may be considered as point sources and all other things remain the same.

$$f \propto \frac{1}{r^2}$$

Specific Inductive Capacity.—Experimentally it is found that the force exerted between two charges at the same distance is different for different media. Therefore the force exerted between two charges depends upon some property of the medium in which they are placed. That property of the medium to which the force is inversely proportional, other things being kept equal, is called the *specific inductive capacity* of the medium. It is represented by the symbol κ.

$$f \propto \frac{1}{\kappa}$$

Coulomb's Law for Electrostatic Charges.—Collecting these separate statements:

$$f = k\frac{1}{\kappa} \cdot \frac{q \cdot q'}{r^2}$$

In which q = magnitude of one charge.
q' = magnitude of the other charge.
r = distance between the two charges.

κ = specific inductive capacity of the medium surrounding the charges.
k = constant of proportionality.

The *unit of quantity* is called the electrostatic unit of quantity if the unit of specific inductive capacity is that of air; the unit of length, the centimeter; the unit of force, the dyne; and the constant, k, is equal to one. In electrostatic units (e.s.u.) Coulomb's law for point sources may be stated as follows:

$$f = \frac{1}{\kappa} \cdot \frac{q \cdot q'}{r^2} \tag{100}$$

As an *illustration, unit quantity,* in the electrostatic system, is a charge of such magnitude that it exerts a force of one dyne on an equal charge placed at a distance of one centimeter in air.

If q and q' are of the same sign their product is positive and they repel each other; if they are of opposite sign their product is negative and they attract each other.

Electrostatic Field.—The region about an electric charge considered with reference to the force exerted by this charge upon another electrostatic charge, is called an *electrostatic field.* The region about one or more charges considered with reference to the force they exert upon a charge, is called the field due to these charges. *Intensity of the electrostatic field* at a point is defined as the ratio of the force exerted upon a charge, placed at that point, to the magnitude of that charge; provided the charge is so small as to leave the field unaltered at all finite distances. It is represented by the symbol $\mathcal{F}$.

$$(101) \qquad \mathcal{F} = \frac{f}{q'}$$

In which q' = charge placed at the point.
f = force exerted on that charge.
$\mathcal{F}$ = intensity of field.

The term "field" is commonly used for "intensity of field." The *unit of intensity of field* is called the electrostatic unit of intensity if the unit of force is the dyne and the unit of charge is the electrostatic unit of charge.

Solving the equation above (101) simultaneously with Coulomb's law for electrostatic charges (100) the expression for the field due to a single charge, q, at a distance, r, is:

$$(102) \qquad \mathcal{F} = \frac{1}{\kappa} \cdot \frac{q}{r^2} \quad \text{From (101) and (100).}$$

$\mathcal{F}$ is a vector having the same direction as r. If q is positive the direction of $\mathcal{F}$ is from q; if q is negative the direction of $\mathcal{F}$ is toward q. $\mathcal{F}$ may be resolved into components along X, Y and Z axes.

$$(103) \qquad \mathcal{F}_x = \mathcal{F} \cos \alpha$$

(104) $$\mathcal{F}_y = \mathcal{F} \cos \beta$$

(105) $$\mathcal{F}_z = \mathcal{F} \cos \gamma$$

In which $\cos \alpha$, $\cos \beta$ and $\cos \gamma$ are the direction cosines of $\mathcal{F}$.

(106) $$\mathcal{F} = \sqrt{\mathcal{F}_x^2 + \mathcal{F}_y^2 + \mathcal{F}_z^2}$$

Resultant Field.—The resultant field, at any point, due to n charges is the vector sum of the n separate intensities. If the charges are numbered from 1 to n, the intensity:

(107) $$\mathcal{F}_k = \frac{1}{\kappa} \frac{q_k}{r_k^2}$$

As k varies from 1 to n:

(108) $$\mathcal{F}_x = \frac{1}{\kappa} \sum_{k=1}^{n} \frac{q_k}{r_k^2} \cdot \cos \alpha_k$$

(109) $$\mathcal{F}_y = \frac{1}{\kappa} \sum_{k=1}^{n} \frac{q_k}{r_k^2} \cos \beta_k$$

(110) $$\mathcal{F}_z = \frac{1}{\kappa} \sum_{k=1}^{n} \frac{q_k}{r_k^2} \cos \gamma_k$$

In which $\mathcal{F}_x$, $\mathcal{F}_y$ and $\mathcal{F}_z$ are the X, Y and Z components of the resultant intensity $\mathcal{F}_r$.

(111) $$\mathcal{F}_r = \sqrt{\mathcal{F}_x^2 + \mathcal{F}_y^2 + \mathcal{F}_z^2}$$

(112) $$\cos \alpha = \frac{\mathcal{F}_x}{\mathcal{F}_r}$$

(113) $$\cos \beta = \frac{\mathcal{F}_y}{\mathcal{F}_r}$$

(114) $$\cos \gamma = \frac{\mathcal{F}_z}{\mathcal{F}_r}$$

The resultant intensity due to any number of charges, distributed in any way, can be completely determined in magnitude and direction, at any point with reference to which the charges may be considered as point sources.

If the n charges lie in the same plane and the resultant intensity is to be found at some point in that plane:

(115) $$\mathcal{F}_r = \sqrt{\mathcal{F}_x^2 + \mathcal{F}_y^2}$$

$$\cos \gamma = 0$$

$$\alpha + \beta = \frac{\pi}{2}$$

(116) $$\therefore \quad \cos \beta = \sin \alpha$$

(117) $$\therefore \quad \tan \alpha = \frac{\mathcal{F}_y}{\mathcal{F}_x}$$ From (112), (113), (116).

If in any region the intensity of field is everywhere the same in magnitude and direction, the field in that region is said to be *uniform*.

Electrification.—That the process of electrification consists of a separation of charges equal in magnitude and opposite in sign, may be shown by means of a rod of hard rubber, a flannel cap, and an electroscope. (This experiment is due to Faraday.) Test the rod and the cap separately by the electroscope and see that they are uncharged. Rub the rod with the cap and, without removing the cap, test again. No charge will be indicated by the electroscope. Remove the cap and test each separately. They will be found to be charged oppositely. If the cap is replaced on the rod the electroscope will indicate no charge, showing that the charges are equal. In a similar way, wherever the process can be traced, every electrification consists of a separation of equal and opposite charges. From the above experiment we infer that each of the uncharged bodies, before rubbing, is uncharged in the same sense as the two bodies together after rubbing; or, that in an uncharged body there is an indefinite quantity of electricity of both kinds, equal in magnitude and similar in distribution.

Conductors and Non-conductors.—Substances were first classified by Gilbert, as electrics and non-electrics but it was later discovered that if one of the so-called electrics was supported by an electric, it also could be electrified. The essential difference is that the electrification, in the case of the electric, remains local whereas in the case of the non-electric it appears in all parts. A better classification is "non-conductors and conductors." Theoretically a conductor is a substance that offers no static resistance to the distribution of the charge, or, when the charge is in equilibrium, the only forces opposing the resultant electric forces are those due to the non-conducting material surrounding it.

Conductors.	*Non-conductors.*
Metals,	Air,
Most liquids,	Amber,
Earth,	Glass,
Human body,	Porcelain,
etc.	Hard rubber,
	Silk,
	Sealing wax,
	Shellac,
	Dry wood,
	etc.

Charging by Induction.—If an uncharged conductor, composed of two parts, is placed in an electrostatic field (Fig. 12) and the connecting wire is broken while

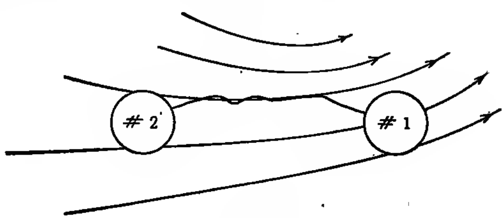


Fig. 12.

in the field, it will be found on testing, that the part No. 1, in the positive direction of the field, will be charged positively and No. 2 negatively. If the two parts are then brought in contact and tested by an electroscope, the combination indicates no charge.

The algebraic sum of the charges must therefore be equal to zero.

In charging by induction equal quantities of positive and negative electricity are separated. These are called induced charges. Since the conducting material offers no static resistance to the distribution of the charge, the separation takes place until at every point inside the conducting material the resultant intensity of field is equal to zero, *i. e.*, the field due to the induced or separated charges is equal and opposite to the inducing field.

From the preceding discussion it follows that:

1. On conducting material, a charge in equilibrium will be distributed upon the surface.

2. The intensity of the field inside the conducting material will be everywhere equal to zero.

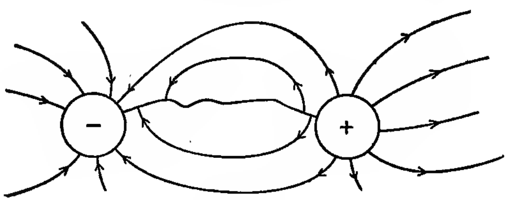


FIG. 13.

3. On the outside, just at the surface of the conductor, the direction of the resultant intensity will be normal to the surface.

Gold-leaf Electroscope.—This form of electroscope consists essentially of two gold leaves suspended side by side from a conductor supported by a non-conducting container. It may be charged by induction, by the following process:

First, touch the disk with the finger. See No. 1, Fig. 14. The leaves, the hand and the earth form one conductor.

Second, while touching it bring a charged body near. No. 2, Fig. 14.

Third, while the rod is held near, remove the finger, thus breaking the conductor into two parts. No. 3, Fig. 14.

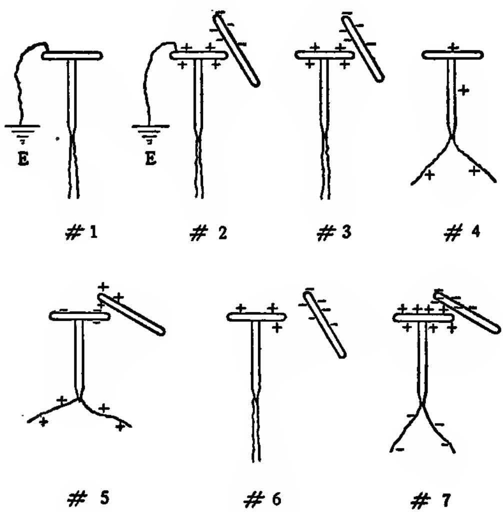


Fig. 14.

Fourth, remove the rod. The charge on the electroscope readjusts itself and the leaves repel each other. No. 4, Fig. 14. If a positively charged body is brought near the positively charged electroscope the leaves merely diverge farther. No. 5, Fig. 14. If a negatively charged body is brought near the positively charged electroscope the leaves first collapse. No. 6, Fig. 14. The positive charge is attracted to the disk and the leaves gradually come nearer together. At a definite position of the rod all the excess charge is attracted to the disk and the leaves hang together. If the negatively charged rod is brought nearer, an excess of negative is repelled to the leaves and they again diverge. No. 7, Fig. 14.

Difference of Potential.—If a charge, q', is placed in an electrostatic field there will be a force, f, exerted upon the charge.

$$f = \mathscr{F}q' \tag{101}$$

If the charge is moved from one point to another in the field work will be done. The ratio of the work done in moving a charge from one point to another, to the magnitude of the charge, is called the *difference of potential* between the two points, provided the charge moved, q, is so small as to leave the distribution of the field unaltered at all finite distances. Difference of potential is represented by the symbol e.

(118) $$e = \frac{w}{q'}$$

From mechanics

$$w = -fs \cos \alpha$$

In which w = work done.
f = force.
s = distance.
α = angle between the positive direction of f and s.

The distance s must be so short that f and α have everywhere the same magnitude. Divide the last equation by q' and combine with (118) and (101):

(119) $$e = -\mathcal{F}s \cos \alpha$$

If

$$\alpha = \frac{\pi}{2}$$

$$\cos \alpha = 0$$

(120) $$\therefore \quad e = 0$$

∴ In moving in a surface everywhere perpendicular to the direction of the intensity no work is done. Such a surface is called an *equi-potential surface.* From this and the property of a charged conductor it follows that the *surface of a conductor* is an equi-potential surface. In going from any point on one equi-potential surface to any point on a second, by any path, the work done is equal to the work done in going from one surface to another in the direction of the intensity.

Potential Theorem.—The expression for the difference of potential between two points in a field due to a single charge, q, is called the potential theorem.

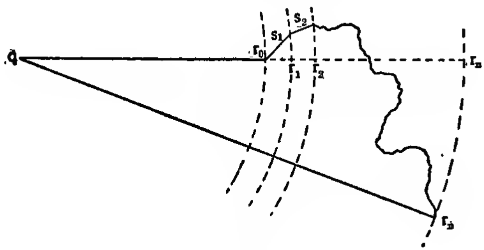


FIG. 15.

Let q = charge producing the field.
r_0 = distance from q to any point.
r_n = distance from q to any other point.
e = difference of potential between r_n and r_0.

The direction of the intensity will be radial and the equipotential surfaces will be concentric spherical surfaces about q. Divide the distance $(r_n - r_0)$ into a large number, n, parts.

Let

(121) $$e = e_1 + e_2 + e_3 + \cdots + e_n$$

In which e_1 = difference of potential between r_1 and r_0,
e_2 = difference of potential between r_2 and r_1,
etc.
e_n = difference of potential between r_n and r_{n-1}.

Let $\mathcal{F} =$ average intensity of field between r_0 and r_1, *i. e.*, that intensity which, if uniform between r_0 and r_1 would give the same difference of potential between the two points.

$$e_1 = - \mathcal{F} s_1 \cos \alpha_1 \qquad (119)$$

(122) $$- s_1 \cos \alpha_1 = (r_1 - r_0) \qquad \text{From Fig. 15.}$$

As

$$r_0 \doteq r_1$$

(123) $$\therefore \quad \mathcal{F} \doteq \frac{1}{\kappa}\frac{q}{r_0^2} \doteq \frac{1}{\kappa}\frac{q}{r_1^2} \doteq \frac{1}{\kappa}\frac{q}{r_1 r_0}$$

(124) $$e_1 \doteq \frac{1}{\kappa}\frac{q}{r_0 r_1}(r_1 - r_0)$$

From (119), (122) and (123).

(125) $$e_1 \doteq \frac{1}{\kappa} q \left(\frac{1}{r_0} - \frac{1}{r_1}\right)$$

Similarly

$$e_2 \doteq \frac{1}{\kappa} q \left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

etc.

$$e_n \doteq \frac{1}{\kappa} q \left(\frac{1}{r_{n-1}} - \frac{1}{r_n}\right)$$

Adding

$$e \doteq \frac{1}{\kappa} q \left(\frac{1}{r_0} - \frac{1}{r_n}\right)$$

This expression is independent of n. The value of n may be taken indefinitely large or the distance indefinitely small.

(126) $$e = \frac{q}{\kappa}\left(\frac{1}{r_0} - \frac{1}{r_n}\right)$$

Potential.—The *potential at a point* in an electrostatic field is the ratio of the work done in bringing a charge from an infinite distance to the point, to the magnitude of that charge (provided the charge is so small as to leave the field unaltered at all finite distances). Potential is represented by the symbol V. It is a scalar quantity.

As $$r_n \doteq \infty$$

$$e \doteq V_0 \qquad \text{From (126).}$$

(127) $$V_0 = \frac{q}{\kappa} \cdot \frac{1}{r_0}$$

In which q = the charge producing the field.
r_0 = distance from q to the point.
V_0 = potential at the point.
κ = specific inductive capacity of the medium surrounding the charge.

Difference of Potentials.—For any point r_1 cm., from a charge q, we have:

$$V_1 = \frac{q}{\kappa} \cdot \frac{1}{r_1} \qquad \text{From (127).}$$

For any other point, r_2:

$$V_2 = \frac{q}{\kappa} \cdot \frac{1}{r_2^2}$$

Subtracting,

(128) $$V_1 - V_2 = \frac{q}{\kappa}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

(129) $$e = V_1 - V_2$$

From (128) and (126).

The difference of potential between two points is equal to the difference of the potentials of the two points.

If in equation (128) $q > 0$ and $r_2 > r_1$

$$V_1 - V_2 > 0$$

In such a case V_1 is said to be "higher" than V_2 instead of "greater" than V_2. If $q > 0$ the intensity is away from q. The direction of the intensity is therefore from the "higher" to the "lower" equi-potential surface.

PROBLEMS.

28. A square measures 10 cm. on a side. Charges of +8 e.s.u., $-8\sqrt{2}$ e.s.u., and − 2 e.s.u., are placed in consecutive corners, Find the magnitude and direction of the intensity of field at the fourth "corner."

What would be the force acting on a charge of + 20 e.s.u., placed at the fourth "corner."

Ans. 0 072 dyne per unit charge.
$\tan^{-1}$ (1.5).
1.44 dynes.

29. An equilateral triangle is inscribed in a circle of radius 10 cm. Charges of +50 e.s.u. each are placed at two of the vertices and a charge of +25 e.s.u. is placed at the third vertex. Find the force on a charge of +20 e.s.u. placed at the center of the circle. Ans. 5 dynes toward the charge of 25 e.s.u.

30. Charges of +20 and +10 e.s.u. are placed at two vertices of an equilateral triangle (10 cm. on a side). Find the intensity of field at the third vertex. If a charge of +8 e.s.u. is placed at the third vertex what force will act on it?

Ans. 0.264 dyne per unit charge.
2.112 dynes.

31. Two positive charges of +80 e.s.u. each are at adjacent corners of a square of 20 cm., side, in air. Find the magnitude and direction of intensity of field at a third vertex, and at the center.

Ans. 0.280 dyne per unit charge.
$\tan^{-1}$ (0.261).
0.56 dyne per unit charge.
Parallel to a side passing through only one of the charges.

32. A charged pith-ball of negligible size is fastened to a perfectly smooth insulating plane of slope $\sin^{-1} 0.015$. Directly up the plane at a distance of 5 cm. a pith-ball of mass 0.5 gram, charged with + 21 e.s.u. is just kept from sliding down. What is the charge on the fixed ball? Ans. +8.75 e.s.u. of charge.

33. The difference of potential between two points 25 cm. and 50 cm. from a charge in air, is found to be 4 ergs per unit charge. What is the difference of potential between points 100 cm. and 200 cm. from the same charge? Ans. 1 erg per unit charge.

34. A charge of 27 e.s.u. is surrounded by air. At a given point the intensity of field is 3 dynes per unit charge. What is the potential at this point? Ans. 9 ergs per unit charge.

35. An equilateral triangle is inscribed in a circle of radius 10 cm. Charges of 25 e.s.u. are placed at the vertices. What is the intensity of field at the center and what is the potential at the same point if the medium is air?

Ans 0 dyne per unit charge.
7.5 ergs per unit charge.

36. Charges of 60 e.s.u. and of 100 e.s.u. are placed at adjacent "corners" on the shorter side of a rectangle, the sides of which are 4 cm. and 3 cm. long. How much work is necessary to move a charge of 15 e.s.u. from the third "corner" to the fourth, the charge moving from the higher to the lower potential?

Ans. −105 ergs of work done on the charge.

37. Two charges of +40 and −24 e.s.u., respectively, are 8 cm. apart in air. Find:

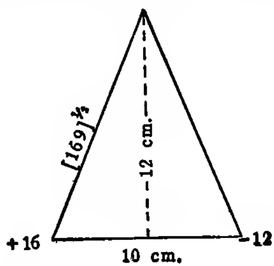
(*a*) Force between them.
(*b*) Intensity of field at a point midway between them.
(c) Potential at the same point.

(*d*) Difference of potential between the middle point and a point 1 cm. from it in the direction of the charge of -24 units.

Ans. 15 dynes.
4 dynes per unit charge.
4 ergs per unit charge.
4 ergs per unit charge.

38. At the extremities of the base of an isosceles triangle are charges of $+16$ and -12 e.s.u. The altitude is 12 cm. and the base is 10 cm. in length. Find the difference of potential between the apex and the middle point of the base.

Let V' = potential at the apex.
V'' = potential at the middle point of the base.



$$V = \frac{q}{r} \qquad \text{In air.} \qquad (127)$$

$$\therefore \quad V' = \frac{16}{\sqrt{169}} - \frac{12}{\sqrt{169}} = +\,0.308$$

Potential is a scalar quantity and may be added algebraically.

$$V'' = \frac{16}{5} - \frac{12}{5} = +\,0.8$$

$$e = V' - V'' \qquad (129)$$

$\therefore \quad e = +\,0.492$ erg per unit charge.

CHAPTER IV.

ELECTROSTATIC INDUCTION.

The product of the specific inductive capacity times the intensity of an electrostatic field, at a point, is called the electrostatic induction at that point. It is represented by the symbol $\mathcal{H}$.

$$\mathcal{H} = \kappa \mathcal{F} \tag{130}$$

In which $\mathcal{F}$ = intensity of field at the point in the medium.

κ = specific inductive capacity of the medium.

$\mathcal{H}$ = electrostatic induction at the point.

The electrostatic induction is a vector quantity having the same direction as the intensity and differing from it in magnitude if the specific inductive capacity is not that of air.

The induction due to a single charge, q, is:

$$\mathcal{H} = \frac{q}{r^2} \quad \text{From (102) and (130).} \tag{131}$$

The induction may be resolved into components along X, Y and Z axes. See equations (103) to (117).

$$\mathcal{H}_X = \kappa \mathcal{F}_x \qquad \text{From (130).} \tag{132}$$

(133) $\mathfrak{D}_x = \sum_{k=1}^{n} \frac{q_k}{r_k^2} \cos \alpha_k$ From (132) and (108).

Similar expressions give the values for the Y and the Z components. The electrostatic induction due to any number of charges is independent of the specific inductive capacity and depends only upon the magnitude and distribution of the charges.

Normal induction over an element of surface is defined as the product of the normal component of the induction times the area of the element. It is represented by the symbol, N.

(134) $$\mathrm{N} = \mathfrak{D} \cdot S \cdot \cos \alpha$$

In which α = angle between $\mathfrak{D}$ and the normal to the surface.

S = an element of surface so small that $\mathfrak{D}$ and α are constant.

The *total normal induction* over a surface is defined as the summation of the normal induction over the elements of which the surface is composed. It is represented by the symbol, N.

(135) $$N = \Sigma \mathrm{N}$$

(136) $$\therefore \quad N = \Sigma \mathfrak{D} \cdot S \cdot \cos \alpha$$

The total normal induction, N, is a scalar quantity. It is independent of the specific inductive capacity of the medium and depends only upon the magnitude and distribution of the electrostatic charges.

Gauss' Theorem.—(*a*) *Charges Inside a Closed Surface.* Let *abc*, Fig. 16, represent the projection of any completely closed surface on a plane.

Let q = a charge at any point within the closed surface.

P = any point on the surface.

S = an element of surface, at P, cut out by a solid angle with apex at q_1.

r = distance from q_1 to P.

PQ = normal to S.

α_1 = angle between the positive direction of the induction, $\mathfrak{N}$, and the positive direction of the normal, PQ. The positive direction of the normal is, by convention, taken outward.

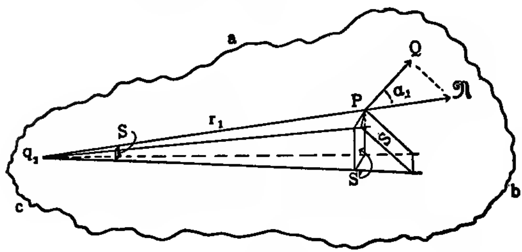


FIG. 16.

$$N_1 = \mathfrak{N}_1 S \cdot \cos \alpha_1 \qquad \text{From (134).}$$

$$N_1 = \frac{q_1}{r_1^2} S \cdot \cos \alpha_1 \qquad \text{From (131).}$$

Let S' be an element of surface cut out on a sphere of radius r_1 and center q_1, by the same solid angle as S. For elements of surface,

$$S' = S \cdot \cos \alpha$$

$$\therefore \quad \mathrm{N}_1 = q_1 \frac{S'}{r_1^2}$$

Let S'' be an element of surface cut out on a unit sphere, with center q_1, by the same solid angle as S and S'.

$$S'' = \frac{S'}{r_1^2}$$

$$\therefore \quad \mathrm{N}_1 = q_1 \cdot S''$$

$$\therefore \quad N_1 = q_1 \Sigma S''$$

$$\Sigma S'' = 4\pi \qquad \text{For a unit sphere.}$$

$$N_1 = 4\pi \cdot q_1$$

The total normal induction over a closed surface, due to a single charge q_1, is equal to 4π times the quantity of the charge enclosed by the surface.

Let

$$N = N_1 + N_2 + N_3 + \cdots + N_n$$

In which

$$N_1 = 4\pi q_1$$

$$N_2 = 4\pi q_2$$

etc.

$$N = 4\pi(q_1 + q_2 + q_3 + \cdots + q_n)$$

Let

$$q = q_1 + q_2 + q_3 + \cdots + q_n)$$

(137) $$N = 4\pi q$$

In which q = the algebraic sum of the charges enclosed by the surface.

N = the total normal induction over the surface.

The total normal induction over any closed surface, due to any number of charges inside, is equal to 4π times the algebraic sum of the charges enclosed.

(*b*) *Charges on the Outside.* Let q' be any charge at a point outside the closed surface. A solid angle will intercept the closed surface at S_1 and S_2 (Fig. 17), or some even number of elements of surface. For each solid angle there will therefore be two elements of surface, S_1 and S_2. Let N' = normal induction through these elements.

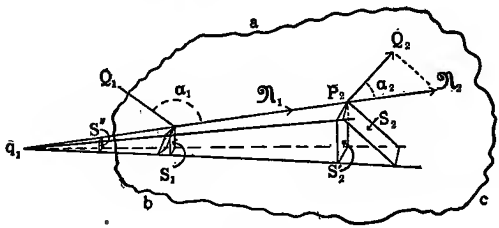


Fig. 17.

$$N_1 = \mathfrak{N}_1 S_1 \cdot \cos\alpha_1 + \mathfrak{N}_2 S_2 \cos\alpha_2$$

Proceeding as before

$$\mathcal{D}_1 S_1 \cdot \cos \alpha_1 = -\, q' S''$$

$$\mathcal{D}_2 S_2 \cdot \cos \alpha_2 = q' S''$$

$$N_1 = 0$$

(138) $$N' = 0$$ From (135).

The induction due to any charge on the outside is equal to zero. The total normal induction over any closed surface is equal to 4π times the algebraic sum of the charges on the inside and is independent of the magnitude and distribution of the charges on the outside.

Application of Gauss' Theorem.—In the application of this theorem to the demonstration of other propositions an imaginary closed surface is chosen in such a manner that part of the surface is parallel to the direction of the induction, part is perpendicular to it and so small that the induction over it is constant.

(139) $$N = N' + N''$$ From (135).

In which N' is the normal induction over that part of the surface which is parallel to the direction of the induction.

(140) $$N' = 0$$ From (136).

In (139), N'' is the normal induction over that part of the surface which is normal to the induction.

(141) $$N'' = \Sigma \mathcal{D} \cdot S$$ From (136).

(142) $$\therefore \quad N = \Sigma \mathfrak{N} \cdot S$$

From (139), (140) and (141).

If the induction is constant over that part of the surface to which it is normal, we may let:

$$\Sigma S = S$$

In which S is the entire surface, normal to the induction.

(143) $$N = \mathfrak{N} \cdot S$$

Field Due to a Uniformly Charged Wire of Infinite Length.—Let ab (Fig. 18) represent a portion of an infinite straight wire.

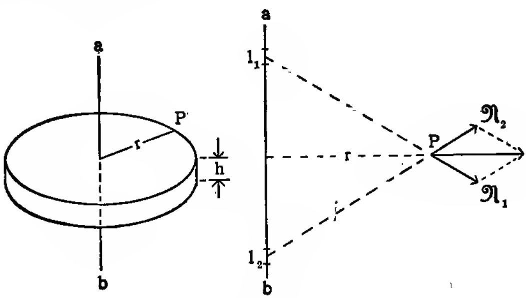


FIG. 18.

Let ρ = charge per unit length of wire.

P be any point at which the intensity is to be found.

r = perpendicular distance from P to the wire.

l_1 and l_2 are two equal elements of length at equal distances from P. $\mathcal{H}_1$ and $\mathcal{H}_2$ are the inductions due to the charges upon l_1 and l_2. They are equal in magnitude.

From the symmetry of the construction, the resultant of these two inductions will be perpendicular to the wire. The resultant induction due to all the elements taken in pairs will be perpendicular to the wire.

Imagine a cylinder of height h and radius r (Fig. 18), placed with ab as its axis. Then the plane surfaces will be parallel to the direction of the induction. The cylindrical part of the surface is everywhere perpendicular to the induction.

$$N = \mathcal{H}\cdot S \qquad \text{From (143).}$$

$$\mathcal{H} = \kappa\mathcal{F} \qquad \text{From (130).}$$

$$S = 2\pi r h$$

(144) $$N = \kappa\mathcal{F}2\pi r h$$

$$N = 4\pi q \qquad \text{By Gauss' theorem.}$$

$$q = \rho\cdot h \qquad \text{By definition of } \rho.$$

(145) $$N = 4\pi\cdot\rho h$$

(146) $$\mathcal{F} = \frac{1}{\kappa}\cdot\frac{2\cdot\rho}{r} \qquad \text{From (144) and (145).}$$

(147) $$\mathcal{F} = \frac{2\cdot\rho}{r} \qquad \text{In air.}$$

Field Outside a Closed Conductor.—Let *abc*, Fig. 19, be a section of a conductor. Imagine an elemen-

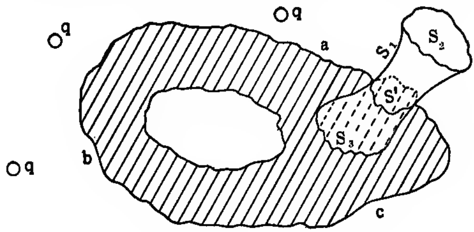


Fig. 19.

tary closed surface of such a form that the curved portion on the outside, S_1, has the same direction as the intensity or induction, and the outer end, S_2, is perpendicular to the intensity. The imaginary surface may have any form, S_3, inside the conducting material, since the intensity is equal to zero. Let S' be that part of *abc* which is inside the imaginary surface.

Let σ = surface density of the charge on S.

N_1 = normal induction over S_1.

N_2 = normal induction over S_2.

N_3 = normal induction over S_3.

$$N = N_1 + N_2 + N_3 \qquad \text{From (135).}$$

$$N_1 = 0 \qquad \text{From (140).}$$

$$N_2 = \mathfrak{N} \cdot S_2 \qquad \text{From (143).}$$

$$\mathfrak{N} = \kappa \cdot \mathfrak{F} \qquad \text{From (130).}$$

$$N_2 = \kappa \cdot \mathcal{F} \cdot S_2$$

$$N_3 = 0$$

$$N = \kappa \cdot \mathcal{F} \cdot S_2$$

$$N = 4\pi q \qquad \text{By Gauss' theorem.}$$

Since S' is taken so small that the charge may be considered uniformly distributed over it, the surface density, σ, may be considered constant.

$$\sigma = \frac{q}{S'}$$

As the outer part approaches the surface of the conductor, S approaches S'.

(148) $$\therefore \quad \mathcal{F} = \frac{4\pi}{\kappa} \cdot \sigma$$

(149) $$\mathcal{F} = 4\pi\sigma \qquad \text{In air.}$$

The intensity of field, at the surface, on the outside of a closed conductor, is equal to 4π times the surface density at that point on the surface and is independent of the magnitude and distribution of all other charges within or without the conductor.

Field Inside a Closed Conductor, Containing No Charge Inside.—On the outside, just at the surface of a conductor, the direction of the intensity is normal to the surface. The surface of a conductor is therefore an equipotential surface. Let *abc*, Fig. 20, be a section of the conductor. Let V_0 be the potential at the surface. V = potential inside the surface.

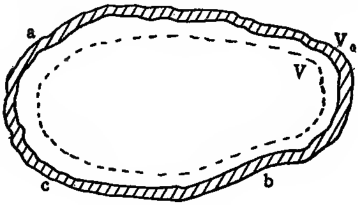


FIG. 20.

(1st) If $V - V_0 > 0$. Imagine a closed surface just inside the surface of the conductor. If the potential inside is greater than outside the direction of the induction is outward and the normal induction is everywhere positive.

$$N > 0$$

$$N = 4\pi q \quad \text{By Gauss' theorem.}$$

$$q = 0 \quad \text{By hypothesis.}$$

(150) $$V - V_0 \not> 0$$

(2d) If $V - V_0 < 0$ the induction is inward and the normal induction is everywhere negative.

$$N < 0$$

$$N = 4\pi q \quad \text{By Gauss' theorem.}$$

$$q = 0 \quad \text{By hypothesis.}$$

(151) $$V - V_0 \not< 0$$

(152) $$\therefore \quad V - V_0 = 0$$

From (150) and (151).

(153) $\therefore \quad \mathcal{F} = 0$ From (152) and definitions of potential and work.

Field Due to an Infinite Uniformly Charged Plane. Let *abcd*, Fig. 21, represent a portion of an infinite

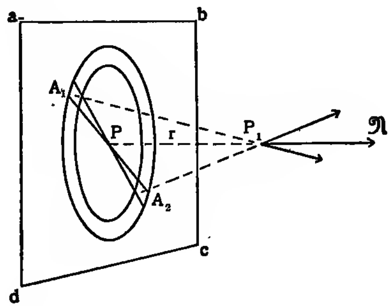


FIG. 21.

plane. (*a*) Let σ = charge per unit area, *i. e.*, surface density of charge.

Let P_1 be any point.

r = distance from P_1 to the plane.

P be the point where r intersects the plane.

With P as a center draw two circles having radii which differ by an infinitesimal length. Draw two diameters forming an angle of infinitesimal magnitude. Between these diameters and the circles there will be two elements of surface A_1 and A_2, which are equal in area. If the plane is uniformly charged the quantity of charge on the two will be equal. The charges on A_1 and A_2 produce inductions at P_1 and from the symmetry of the construction, the resultant will be perpendicular to the plane. All the elements of area lying between the two circles can be thus arranged in pairs diametrically opposite each other. The resultant induction, due to all the charges on all elements of surface, at equal distance from P_1, will be perpendicular to the plane. The resultant induction at P_1, due to all concentric rings, will be perpendicular to the plane. The resultant induction, due to an infinite plane uniformly charged, is perpendicular to the plane.

In a similar way it can be shown that the induction on the opposite side of the plane is perpendicular to it. From the symmetry of the construction, the magnitude of the induction is the same at symmetrically placed points on opposite sides of the plane.

(*b*) Imagine a cylinder of length $2r$ (Fig. 22), cross-

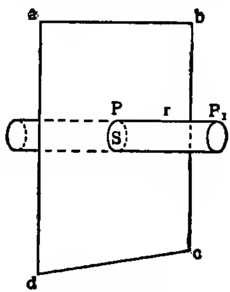


FIG. 22.

section S, and with the same plane, *abcd*, perpendicular to its axis at the middle point. The cylindrical part of the surface will be parallel to the direction of the induction. The plane surfaces are taken perpendicular to the induction. The cylinder will intercept a surface on *abcd* equal to S.

Let q = quantity of charge on S.
σ_1 = surface density on one side.
σ_2 = surface density on other side.

$$N = \Sigma \mathfrak{N} S \qquad \text{From (139), (140) and (141).}$$

$$\Sigma S = 2S$$

$$N = 2\mathfrak{N} \cdot S$$

$$\mathfrak{N} \cdot = \kappa \cdot \mathcal{F} \qquad \text{From (130).}$$

$$N = 2S \cdot \kappa \cdot \mathcal{F}$$

$$N = 4\pi q \qquad \text{By Gauss' theorem.}$$

$$q = S(\sigma_1 + \sigma_2) \qquad \text{By definition.}$$

From symmetry

$$\sigma_1 = \sigma_2$$

(154)
$$\mathcal{F} = \frac{4\pi\sigma_1}{\kappa}$$

If

$$\sigma = \sigma_1 + \sigma_2$$

i. e., σ = total charge on both sides per unit area of one side.

(155)
$$\mathcal{F} = \frac{2\pi\sigma}{\kappa}$$

(156)
$$\mathcal{F} = 2\pi\sigma \qquad \text{In air.}$$

If σ is positive the direction of the induction and of the intensity is away from the plane; if σ is negative the direction is toward the plane. The intensity and the induction are independent of the distance of the point from the plane provided the distance is small compared with the dimensions of the plane.

Field Due to Two Infinite Planes Oppositely Charged to the Same Uniform Surface Density.— Let *ab* and *cd*, Fig. 23, be parts of two planes perpendicular to the X axis.

Let $\mathscr{F}_1$ = intensity due to charge on *ab*.

$\mathscr{F}_2$ = intensity due to charge on *cd*.

Taking the positive direction of intensity as the positive direction of the X axis:

$$\mathscr{F} = \mathscr{F}_1 + \mathscr{F}_2 \text{ for any point.}$$

Between the two planes:

$$\mathscr{F}_1 = \frac{2\pi\sigma}{\kappa}$$

$$\mathscr{F}_2 = \frac{+\,2\pi\sigma}{\kappa}$$

(157) $$\therefore \quad \mathscr{F} = \frac{4\pi\sigma}{\kappa}$$

For any point on the outside:

$$\mathscr{F}_1 = \pm \frac{2\pi\sigma}{\kappa}$$

$$\mathscr{F}_2 = \frac{\mp\,2\pi\sigma}{\kappa}$$

(158) $$\mathscr{F} = 0$$

The direction of the intensity between the two planes

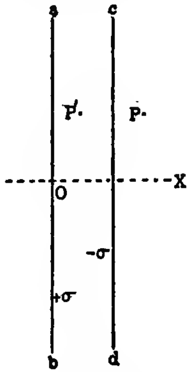


FIG. 23.

is from the plane which is charged positively to the one charged negatively.

Field Due to a Uniformly Charged Sphere.—Let *ABC*, Fig. 24, represent a sphere of radius a.

q_s = charge uniformly distributed over the surface.

P be any point.

r = distance from the center of the sphere to P.

From the conditions of symmetry the direction of the induction is radial or can be shown to be so by the method of concentric rings as in the case of the infinite plane. From symmetry it is also evident that the magnitude of the induction is the same for all points equally distant from the center.

Imagine a sphere of radius r concentric about ABC. The induction over this sphere will be everywhere perpendicular to the surface and equal in magnitude.

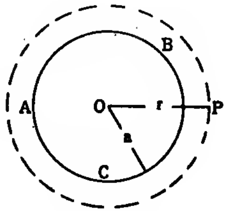


Fig. 24.

$$N = \Sigma \mathcal{H} \cdot S \qquad (141)$$

$$\mathcal{H} = \kappa \cdot \mathcal{F} \qquad (130)$$

$$\Sigma S = 4\pi r^2$$

$$N = \kappa \mathcal{F} 4\pi r^2$$

$$N = 4\pi q \qquad (137)$$

$$\mathcal{F} = \frac{1}{\kappa} \frac{q}{r^2}$$

If $r < a$

$$q = 0$$

(159) $$\mathcal{F} = 0$$

If $r > a$

$$q = q_s$$

(160) $$\mathcal{F} = \frac{1}{\kappa} \frac{q_s}{r^2}$$

Compare this expression with (102). The intensity, due to a uniformly charged sphere, for all points on the outside, is the same as if the charge were at the center and for all points inside is equal to zero.

Charging by Induction from the Inside of a Completely Closed Conductor.—Let *abc*, Fig. 25, be a section of any completely closed hollow conductor which is connected to earth. Let q_1 be a charge placed at some point inside the cavity. Let q_2 be the induced charge on the inner surface of the conductor. The

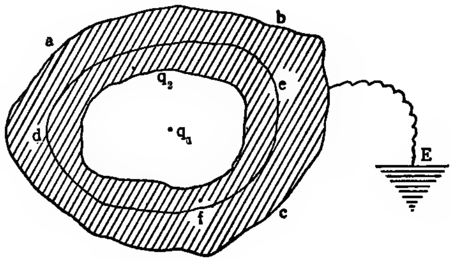


FIG. 25.

intensity at all points inside the conducting material is equal to zero. Imagine a completely closed surface, *def*, lying in the conducting material and enclosing the cavity.

$$N = 0$$

$$N = 4\pi q \qquad (137)$$

$$\therefore \quad q = q_1 + q_2$$

(161) $$\therefore \quad q_1 = -q_2$$

In charging by induction, from the inside, the induced charge is equal in magnitude and opposite in sign to the inducing charge.

Weight Electrometer.—The weight electrometer is an instrument for measuring differences of potential in absolute units. It consists of an equal-arm balance in which the electric force is balanced against the force of gravity. A circular metallic disk, *a*, Fig. 26, is suspended from one arm by insulating threads. It is surrounded by a guard ring, *b*, which is insulated from the fixed plate, *c*. The plate, *a*, and guard ring, *b*, are in metallic contact by means of a very fine flexible wire. The fixed plate, *c*, and the ring, *b*, are connected to a source of difference of potential, *e*. The disks, being oppositely charged, will attract each other and the force of attraction, *f*, is balanced by a counterpoising weight, *mg*.

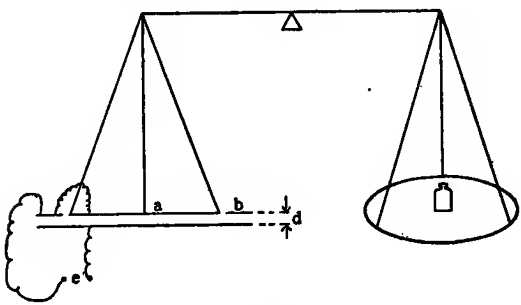


FIG. 26.

Let A = area of the movable disk, a.

d = distance between the planes. (It is small compared with the radius of a.)

q = quantity of charge on A.

σ = surface density.

$\mathcal{F}_1$ = intensity due to lower plate.

$\mathcal{F}$ = intensity due to both plates.

$$f = \mathcal{F}_1 \cdot q \qquad \text{From (101).}$$

In which f = force exerted upon the upper plate. Since d is small

$$\mathcal{F}_1 = 2\pi\sigma \qquad \text{From (156) (in air).}$$

$$q = A\sigma \qquad \text{By definition.}$$

(162) $$\therefore\ f = 2\pi\sigma^2 \cdot A$$

$$e = \mathcal{F}d \qquad \text{By definition of } e \text{ and (156).}$$

$$\mathcal{F} = 4\pi\sigma \qquad \text{In air (157).}$$

(163) $$\therefore\ e = 4\pi\sigma \cdot d$$

(164) $$e = \sqrt{\frac{8\pi d^2}{A} \cdot f} \qquad \text{From (162) and (163).}$$

By the weight-electrometer the value of e is determined in terms of mass, length, time and the specific inductive capacity of air. Such a determination is called an absolute determination and the instrument is called an absolute instrument, *e. g.*, absolute electrometer, absolute tangent galvanometer, magnetometer, etc. In general, quantities can be measured in two ways, viz: either in terms of fundamental quantities or by comparison to some like quantity taken as a standard. The first method is generally difficult and laborious but is necessary to calibrate standards which are used in the latter method. In the quadrant electrometer, an instrument of the second class, the electric force is balanced against the elastic force of a torsion suspension. This permits accurate comparison of much smaller differences of potential.

Verification of Coulomb's Law for Electrostatic Charges.—Since every proposition in electrostatics is based, directly or indirectly, on Coulomb's law for electrostatic charges a rigorous experimental test should be given.

Cavendish's Method.—The apparatus consists of a sensitive electrometer and two concentric insulated metallic spheres. In the outer sphere there is a small opening with a cap, *a*, Fig. 27, supported by a silk thread. Attached to the cap there is a short wire connecting the two spheres when the cap is in position. The outer sphere is charged; the cap is removed; the outer sphere connected to the earth and the inner sphere tested by the electrometer. No charge is indicated. This method was devised by Cavendish and performed by Maxwell and others with great care and accuracy.

$$f = \frac{1}{\kappa}\frac{q \cdot q'}{r^2} \qquad \text{By hypothesis.}$$

Everywhere inside the outer sphere,

$$\mathcal{F} = 0 \qquad \text{By deduction.}$$

but

$$\mathcal{F} = 0 \qquad \text{By experiment.}$$

In the same way every deduction made from Cou-

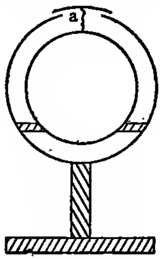


FIG. 27.

lomb's law can be verified within the limits of experimental error.

$$f = \frac{1}{\kappa} \cdot \frac{q \cdot q'}{r^2} \tag{100}$$

PROBLEMS.

39. Three charges of 10, 20, and -30 e.s.u. are situated at three vertices of a right triangle having sides 3 and 4 cm. long. The charge of 10 is at the vertex formed by the short side and by the hypothenuse, and the charge of 20 at the right angle. Find the total normal induction over spheres of radii 2, 4, and 6 cm. drawn about the charge of 10 as a center.

Ans. $N = 40\ \pi$ e.m.u. of induction.
$N = 120\ \pi$ e.m.u. of induction.
$N = 0$ e.m.u. of induction.

40. Four large parallel planes are arranged at intervals of 1 cm. in air. They are charged alternately positively and negatively to surface densities of 4 e.s.u. of quantity per square centimeter. The inner planes are charged on both sides. Find the intensities at points 0.5 cm. from each plane.

Ans. 0 dynes per unit charge.
$16\ \pi$ dynes per unit charge.
$16\ \pi$ dynes per unit charge.
$16\ \pi$ dynes per unit charge.
0 dynes per unit charge

41. A long thin uniformly charged non-conductor is placed parallel to two large uniformly charged non-conducting planes equally distant from the wire and parallel to each other. Find the force per unit length exerted upon the wire, in terms of the surface density on the plane and linear density on the wire. Also find the resultant intensity of field at points half way between the wire and the planes.

Ans. $4\pi\rho\sigma$ dynes.

$$4\pi\sigma - \frac{8\rho}{d} \text{ dynes per unit charge.}$$

$$4\pi\sigma + \frac{8\rho}{d} \text{ dynes per unit charge.}$$

In which d = distance between the planes.

42. The plate of a weight electrometer is 6 cm. in radius. A mass of 15 grams on the pan just holds the plate in equilibrium 4 mm. from the fixed plate. What difference of potential is being measured? Ans. 22.9 ergs per unit charge.

CHAPTER V.

CAPACITY.

If we have two conductors separated by a non-conductor, and joined to a source of difference of potential, they will be charged positively and negatively. The ratio of the positive charge to the difference of potential is called the *capacity* of the combination. Such a combination, when used for its capacity effect, is called a simple *condenser*.

$$C = \frac{q}{e} \tag{200}$$

In which C = capacity of the combination.
q = quantity of charge on the positive plate.
e = difference of potential between the two plates.

If one of these conductors is removed to an infinite distance:

$$e \doteq V$$

In which V = potential of the remaining conductor.

$$\therefore \quad C = \frac{q}{V} \tag{201}$$

is called the capacity of a single conductor.

Unit of Capacity.—The *unit of capacity* is called the electrostatic unit of capacity if the charge is measured in e.s.u. of charge and the difference of potential is measured in e.s.u. of potential difference. As an illustration, a condenser of unit capacity, in the electrostatic system, is one of such magnitude that it would be charged to one e.s.u. of potential difference by one e.s.u. of quantity of charge.

Capacity of a Sphere.—

$$C = \frac{q}{V} \qquad \text{By definition (201).}$$

$$V = \frac{1}{\kappa}\frac{q}{r} \qquad \text{From (127).}$$

(202) $$\therefore \quad C = \kappa r$$

From (202), C depends only on the radius, r, of the sphere and the specific inductive capacity of the medium.

Capacity of Two Concentric Spheres.—Fig. 28 represents two concentric spheres of conducting material, separated by a non-conductor.

Let $r_1 =$ the outer radius of the inner sphere.
$r_2 =$ the inner radius of the outer sphere.
$e =$ the difference of potential maintained by the source to which the spheres are connected. The inner sphere is charged by conduction by means of the insulated wire *a*.

Let $q =$ charge on the inner sphere.

$-q$ = charge on the inner surface of the outer sphere.

$$C = \frac{q}{e} \tag{200}$$

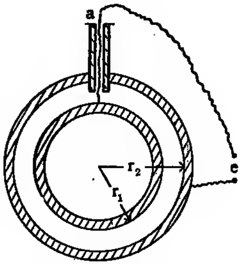


FIG. 28.

If the positive directions of the intensity are taken outward from the sphere:

$$\mathcal{F} = \mathcal{F}_1 + \mathcal{F}_2$$

In which $\mathcal{F}_1$ and $\mathcal{F}_2$ are the intensities due to the charges on the inner and on the outer spheres respectively.

(1st) Within the inner sphere:

$$\mathcal{F}_1 = 0 \qquad \text{From (159).}$$

$$\mathcal{F}_2 = 0 \qquad \text{From (159).}$$

$$\therefore \quad \mathcal{F} = 0 \quad \text{as } r \text{ varies from } 0 \text{ to } r_1.$$

(2d) Between the two spheres:

$$\mathcal{F}_1 = \frac{1}{\kappa}\frac{q}{r^2} \qquad \text{From (160).}$$

$$\mathcal{F}_2 = 0 \qquad \text{From (159).}$$

(203) $$\mathscr{F} = \frac{1}{\kappa}\frac{q}{r^2} \text{ as } r \text{ varies from } r_1 \text{ to } r_2.$$

(3d) Outside both spheres:

$$\mathscr{F}_1 = \frac{1}{\kappa}\frac{q}{r^2} \qquad \text{From (160).}$$

$$\mathscr{F}_2 = -\frac{1}{\kappa}\frac{q}{r^2} \qquad \text{From (160).}$$

$$\mathscr{F} = 0 \text{ as } r \text{ varies from } r_2 \text{ to } \infty.$$

Applying equations (127), (128), (129) and (126) to this case of two concentric spheres:

$$e = \frac{q}{\kappa}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

(204) $$C = \kappa\left(\frac{r_1 r_2}{r_2 - r_1}\right)$$

Again the capacity depends only on the radii and specific inductive capacity of the medium between the two spheres.

Capacity of Two Parallel Plates.—Fig. 29 represents two parallel plates joined to a source of difference of potential, e.

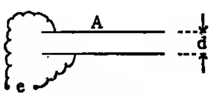


FIG. 29.

Let A = area of one plate.

d = distance between them.

σ = surface density of charge on A.

$$C = \frac{q}{e} \qquad \text{From (200).}$$

$$q = A\sigma \qquad \text{By definition.}$$

If the distance between plates is small compared with the dimensions of the plates the field between them is uniform.

$$e = \mathcal{F}\cdot d \qquad \text{From (119).}$$

$$\mathcal{F} = \frac{4\pi\sigma}{\kappa} \qquad \text{From (157).}$$

$$\therefore \quad e = \frac{4\pi d\sigma}{\kappa}$$

(205)
$$C = \kappa\frac{A}{4\pi d}$$

Again, the capacity, C, depends only on the specific inductive capacity of the medium between the two plates and the geometrical arrangement.

Capacities in Parallel.—If a number of capacities are connected to the same difference of potential, e, they are said to be joined in parallel. In this arrangement all parts are charged by conduction.

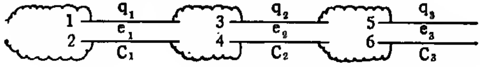


FIG. 30.

Let C = capacity of the combination.
e = difference of potential for the combination.
q = quantity of charge on the positive plates.

$$C = \frac{q}{e} \qquad \text{By definition.}$$

$$C_1 = \frac{q_1}{e_1} \qquad \text{By definition.}$$

$$C_2 = \frac{q_2}{e_2}$$

etc., for as many as are in parallel.

$$q = q_1 + q_2 + q_3 + \cdots$$

$$e = e_1 = e_2 = e_3 = \cdots$$

(206) $$\therefore \quad C = C_1 + C_2 + C_3 + \cdots$$

If

$$C_1 = C_2 = C_3 = \cdots$$

(207) $$C = nC_1$$

In which n = the number of capacities in parallel.

Capacities in Series. — In this arrangement two parts only are connected to the source, e, Fig. 31. These are charged by conduction. The others are charged by induction.

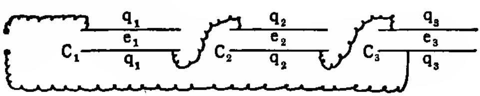


FIG. 31.

Let C = capacity of the combination.
e = difference of potential for the combination.
q = positive charge conducted from e to the combination.

$$C = \frac{q}{e} \quad \text{For the combination.}$$

$$C_1 = \frac{q_1}{e_1} \quad \text{For the first capacity.}$$

$$C_2 = \frac{q_2}{e_2} \quad \text{For the second capacity.}$$

etc., for as many as are in series.

$$e = e_1 + e_2 + e_3 + \cdots$$

$$q_1 = -q_3'$$

$$q_2 = -q_1'$$

$$q_3 = -q_2'$$

etc.

If the distance between the plates in each capacity is small compared with their area,

$$q_1 + q_1' \doteq 0$$

$$q_2 + q_2' \doteq 0$$

$$q_3 + q_3' \doteq 0$$

$$\therefore \quad q \doteq q_1 \doteq q_2 \doteq q_3$$

(208) $$\therefore \quad \frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots$$

If

$$C_1 = C_2 = C_3 \cdots$$

(209) $$C = \frac{1}{n} C_1$$

In which n = the number of capacities in series.

Parallel Plates.—Fig. 32 represents a capacity composed of several parallel plates separated by a non-conductor. Alternate plates are in metallic contact, thus forming a number of capacities in parallel. Since the electrostatic field outside a parallel plate condenser is equal to zero, no work is done in moving one condenser with reference to another. The capacity of the arrangement, shown in Fig. 32, is the same as that shown in Fig. 30. One plate serves the purpose of two. Plate No. 2 coincides with plate No. 4; plate No. 3 coincides with plate No. 5, etc., except for the plates at the ends.

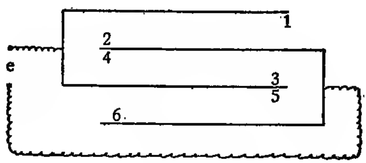


FIG. 32.

Let n = number of plates. $\therefore n - 1$ = the number of capacities in parallel.

$$C = (n - 1)C_1 \qquad \text{From (207).}$$

$$C_1 = \kappa \frac{A}{4\pi d} \qquad \text{From (205).}$$

(210) $$\therefore \quad C = \kappa \frac{(n-1)A}{4\pi d}$$

Comparison of Two Capacities.—Let C_1 and C_2, Fig. 33, represent two capacities. Join C_1 to a source of difference of potential and measure e_1 by an electrometer.

(211) $$C_1 = \frac{q_1}{e_1} \qquad \text{From (200).}$$

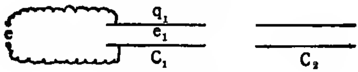


FIG. 33.

Disconnect from the source e and connect to the other capacity C_2 and measure e_2 (Fig. 34).

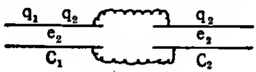


FIG. 34.

(212) $$C_1 = \frac{q_1 - q_2}{e_2} \qquad \text{By definition.}$$

(213) $$C_2 = \frac{q_2}{e_2}$$

Eliminating q_1 and q_2 by use of (211), (212) and (213),

(214) $$\frac{C_1}{C_2} = \frac{e_2}{e_1 - e_2}$$

Determination of Specific Inductive Capacity.—If air is the medium between the plates of C_2 and κ is the specific inductive capacity of the medium between the plates of C_1, the relation will be:

$$C_1 = \kappa C_2 \tag{215}$$

if the geometrical dimensions of the two capacities are the same.

$$\therefore \quad \kappa = \frac{e_2}{e_1 - e_2} \tag{216}$$

Leyden Jar.—A Leyden jar may be considered as an approximate parallel plate condenser, in which:

$$A = \pi r^2 + 2\pi r h. \tag{217}$$

In which r = radius of the jar.
h = height of the tin foil.

Energy of a Condenser.—If the condenser is uncharged and the charge q, which is finally to be found on the plate, is transferred from one plate to the other in n successive parts each of quantity q_1:

$$q = nq_1$$

Let w_1, w_2, etc., = the work done in transferring the respective charges q_1, q_2, etc.
w = work done in charging the condenser to the final difference of potential.

$$w = w_1 + w_2 + w_3 + \cdots + w_N$$

$$w_1 \doteq e_1 q_1 \qquad \text{From (118).}$$

$$w_2 \doteq e_2 q_2 \qquad \text{From (118).}$$

etc.

In which e_1, e_2, etc., represent the successive values of the difference of potential as the condenser is charged.

$$w \doteq q_1(e_1 + e_2 + e_3 + \cdots + e_N) \tag{217}$$

$$w \doteq q\left(\frac{e_1 + e_2 + e_3 + \cdots + e_N}{n}\right) \tag{218}$$

The expression in brackets is the average value of the difference of potential as the charge is transferred. Since at each step in the process

$$e = \frac{1}{C}\cdot q$$

As $n \doteq \infty$

$$\frac{e_1 + e_2 + e_3 + \cdots + e_N}{n} = \frac{0 + e}{2} \tag{219}$$

$$\therefore \quad w = \tfrac{1}{2}\cdot e \cdot q \tag{220}$$

In which w = potential energy of the condenser.
e = final difference of potential.
q = final charge.

PROBLEMS.

44. Two small bodies having capacities of 20 and 30 e.s.u., and potentials of 1.8 and 0.8 e.s.u. respectively, are placed in air 120 cm. apart. If they are connected by a fine straight wire, at what points between the bodies will the potential remain unchanged? What is the potential at these points? What quantities of charge are on the capacities before and after the connection?

Ans. 40 cm. and 90 cm. from capacity 20 e.s.u.
1.2 ergs per unit charge.

24 e.s.u of charge change to 36 e.s.u. on capacity of 30 e.s.u.
36 e.s.u. of charge change to 24 e.s.u. on capacity of 20 e s.u.

45. Find the charge on a sphere of 10 cm. radius, in air, when the potential is 50 e.s.u. Ans. 500 e.s.u. of quantity.

46. One thousand globules of water each of .02 cm. diameter are charged to the same potential. Compare their electrical potential and surface density with that produced when they have united into a single globule.

Ans. The potential is increased to 100 times its former value, and the surface density to 10 times its former value.

47. If a sphere, of 6.4 cm. diameter, in air and charged with 98 e.s.u. of quantity be connected by a long fine wire to a second conducting sphere of radius 2.5 cm., what will be the charge and potential of each sphere and what the capacity of the combination?

Ans. 55 e.s.u. of quantity on sphere 6.4 cm. in diameter.
43 e.s.u. of quantity on sphere 2.5 cm.
17.2 – ergs per unit charge.
5.7 e.s.u. of capacity.

48. If the combination described in the preceding problem be connected to a third sphere of radius 3 cm., what will be the charge and potential on each and what will be the capacity of the combination?

Ans. 36 e.s.u. of quantity on sphere 6.4 cm.
28.2 e.s.u. of quantity on sphere 2.5 cm.
33.8 e.s.u. of quantity on sphere 3 cm.
11.26 ergs per unit charge.
8.7 e.s.u. of capacity.

49. If n spheres are connected by a long fine wire of negligible capacity what will be the capacity and the potential of the combination in terms of the radii and original potential of the spheres?

Ans. $C = \Sigma r$

$$V = \frac{\Sigma r V}{\Sigma r}$$

50. What is the capacity of a spherical condenser in which the external diameter of the inner sphere is 48 cm., and the dielectric is 0.25 cm. thick? The specific inductive capacity of this dielectric is 4.3 times that of air. Ans. 10,000 e s.u. of capacity.

51. One quarter of the earth's circumference is almost $(10)^9$ cm. Compare the capacity of the earth with that of a spherical condenser of two concentric spheres having radii of 100 cm. and 100.1 cm. The substance between the spheres is glass of specific inductive capacity 6 28 times that of air.

How many of these condensers in parallel would be equivalent to the earth in electrical capacity?

Ans. Capacity of earth $= \frac{2}{\pi}(10)^9$ e.s.u.

Capacity of condenser $= 2\pi(10)^5$ e.s.u.

Number of condensers $= \frac{1}{\pi}(10)^4$.

52. Find the capacity of a plate condenser made of two rectangular conductors 32 cm. long, 22 cm. broad, and 0.2 cm. apart in air. If the air be replaced by 0 2 cm. sheet of glass of specific inductive capacity 7 times that of air, find the charge on one plate when the difference of potential is 20 ergs per unit charge.

Ans. 281 e.s.u. of capacity
39,300 e.s.u. of quantity.

53. Two plate condensers are joined in parallel. One is a 15 plate air condenser, each plate 11 cm. long, 5 cm. wide and 3 mm. apart; the other a mica condenser of 101 plates each 22 cm. long, 15 cm. wide and 0.5 mm. apart. The specific inductive capacity of the mica is 8 times that of air. Find the capacity of the combination. Ans. 421,000 e.s.u. of capacity.

54. A Leyden jar 0.3 cm. thick is 12 cm. in diameter and the tin foil extends 18 cm. high. Find the capacity and quantity of charge on it when it is connected to a difference of potential of 10 ergs per unit charge.

Ans. 1,470 e.s.u. of capacity.
14,700 e.s.u. of quantity of charge.

55. Two condensers, when in series, had a joint capacity of 3 e.s.u. When in parallel the joint capacity was 16 e.s.u. What was the capacity of each taken separately?

Ans. 4 e.s.u. of capacity.
12 e.s.u. of capacity.

56. A pair of circular plates of radii 10 cm. each, are 1 mm. apart in air. They are charged to a difference of potential of 20 ergs per unit charge and are then connected to the plates of an uncharged condenser. The difference of potential falls to 3 ergs per unit charge. Find the capacity of the second condenser.

Ans. 1,420 e.s.u. of capacity.

57. A spherical conductor having a radius of 2.7 cm. is charged to a potential of 125 ergs per unit charge. On being connected by a long fine wire, to a second uncharged conductor the potential fell to 22 ergs per unit charge. What was the capacity of the combination?

Ans. 15.3 e.s.u. of capacity.

58. An air condenser is charged and the difference of potential between its plates measured. It is then connected to a condenser exactly similar except that the dielectric is unknown. The difference of potential falls to 1/4 its former value. What is the specific inductive capacity of the dielectric?

Ans. 3 times that of air.

59. A condenser consists of two circular plates 15 cm. in diameter, separated by a film of air 1.2 cm. thick. When 750 ergs have been expended in charging it, what will be the charge on one plate and the difference of potential between plates?

Ans. 132. e.s.u. of quantity of charge.
11.3 ergs per unit charge.

60. If a spherical air condenser of shells 64 cm. and 60 cm. in diameter be charged to a potential of 500 ergs per unit charge, and afterwards be connected to a similar uncharged condenser having diameters of 64 cm. and 68 cm., how much energy will be lost in the first condenser by the change?

Ans. 4.68 joules.

CHAPTER VI.

ELECTROSTATIC MACHINES.

Electrophorus.—A simple form of static machine is one called the electrophorus. It consists of a plate of non-conducting material such as hard rubber (*a*, Fig. 35) and a metal plate with a non-conducting handle, *b*, Fig. 35. The process consists in the electrification of the hard rubber, by means of cat's fur and then charging the metallic plate by induction. (See gold-leaf electroscope.) We then have a parallel plate condenser. Disconnect from the earth and, as the plates are separated a short distance, the mechanical work done against the electric force is:

$$w = f \cdot d$$

$$f = 2\pi\sigma^2 \cdot A \qquad \text{From (152).}$$

(221) $$w = 2\pi\sigma^2 A d$$

$$e = 4\pi\sigma \cdot d \qquad \text{From (135).}$$

$$q = A\sigma$$

(222) $$w = \tfrac{1}{2} eq$$

Compare this with the general expression for the potential energy of a condenser. See equation (220). Since the charge remains constant the difference of potential is increased as the plates are separated.

Hence the electrophorus is a machine for converting mechanical energy into electrical potential energy.

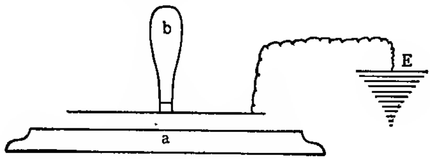


FIG. 35.

Induction Machine.—The Toepler-Holtz machine is a typical form of induction machine. Although it consists of two coaxial circular plates, its action is more easily represented by two concentric cylinders. See Fig. 36. The cylinder, *A*, is fixed and made of non-conducting material. It corresponds to the fixed plate in the machine. *B* and *C* are two armatures of conducting material. They are enlarged at the extremities and narrow at the middle. *E* is a movable cylinder of non-conducting material and corresponds to the movable plate. An even number of disks, *F* and *F′*, of tin foil, with raised buttons are arranged at regular intervals on this cylinder. *G* is an insulated conductor carrying two metallic brushes which come in contact with the buttons just as the disks pass from under the armature. *H* and *H′* represent two brushes which bring the armature into metallic contact with the buttons as they pass under the armature.

Assume that all parts of the machine are discharged and that one of the buttons, *F*, is, by any process, given a small charge, say positive. Rotate the movable cylinder and, as *F* moves under the brush attached to *H*, the armature is charged by conduction until *C* and *F* are at the same potential. As *F* passes under

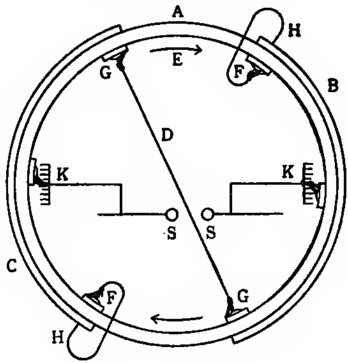


FIG. 36.

the comb, *K*, negative electricity is attracted by *C* and positive is repelled by induction. By the principle of point discharge, which will be explained later, the button is partly discharged. This causes a difference of potential between *K* and *K′* and the potential of *F* is decreased. As *F* moves toward *G* its potential is less than that of *C*. An excess of negative will therefore be attracted by *C* to the disk and an excess of positive repelled by *C* to the button. As *F* comes in contact with *G*, the excess of positive is repelled to *G′* and *F* leaves *G* negatively charged. As the negative charge on F is separated from the positive charge on C, the difference of potential between C and F is increased. If this difference of potential is greater than the difference of potential between C and B, negative charge will pass from F to B as F comes in contact with H'. Thus the difference of potential between C and B is increased. As F passes from H' to H, the same successive variations of difference of potential occur. On this side the button is at the higher and the armature at the lower potential. As this process continues, the difference of potential between K and K' increases until a discharge takes place. This discharge occurs between S and S', called the spark gap, if they are not too far apart. The quantity transferred by such a discharge is:

$$q = C \cdot e \tag{200}$$

In which e = difference of potential between S and S'. It is a function of the distance, as is shown in Fig. 37.

C = capacity connected to S and S'.

The machine transforms mechanical energy into electrical potential energy. When the discharge takes place this energy is converted into other forms of energy, generally heat, light and sound.

The curve, Fig. 37, shows the sparking potential difference (in e.s.u.) between sharp needle points in air. If the radii of S and S' are very small, as in two pointed conductors, the discharge takes place at a lower poten-

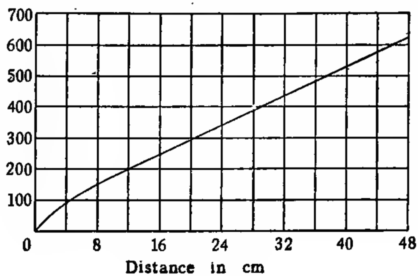


FIG. 37.

tial difference. This is the principle of the comb discharge. It may be illustrated as follows:

Let ab and cd, Fig. 38, represent two spheres connected by a fine wire.

Let V = the potential of the spheres.

q_1 = charge on ab.

q_2 = charge on cd.

$\mathcal{F}_1$ = intensity of field at the surface of ab.

$\mathcal{F}_2$ = intensity of field at the surface of cd.

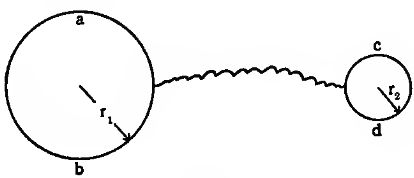


FIG. 38.

$$V = \frac{q_1}{r_1} \tag{117}$$

$$V = \frac{q_2}{r_2} \tag{117}$$

$$\therefore \quad \frac{q_1}{q_2} = \frac{r_1}{r_2}$$

$$\mathcal{F}_1 = \frac{q_1}{r_1^2} \qquad \text{From (150)}$$

Similarly for $\mathcal{F}_2$

(223) $$\frac{\mathcal{F}_1}{\mathcal{F}_2} = \frac{r_2}{r_1}$$

For the same potential the intensity of electrostatic force tending to produce discharge is, roughly, inversely as the radius of curvature. The teeth of the combs are therefore drawn to sharp points. Wherever a discharge is undesirable the conductors are constructed with large radii of curvature.

CHAPTER VII.

ELECTRODYNAMICS.

The phenomena considered under the subject of electrostatics are those conditioned by a conductor all parts of which are at the same potential. In electrodynamics the phenomena considered are those conditioned by a conductor the parts of which are at different potentials.

Sources of Difference of Potential. — A source of difference of potential is a device for converting some other form of energy, *e. g.*, mechanical or chemical, energy into electrical energy. It may be determined and measured by means of an electrometer. The work per unit charge in converting mechanical, chemical or other form of energy into electrical energy is called the "*electro-motive force*" (e.m.f.) of the device.

If a conductor is joined to the terminals of a source of difference of potential, all parts tend to come to the same potential, converting the electrical energy into heat or some other form of energy. Such an arrangement is called an *electric circuit.* If the device maintains the difference of potential it is called a constant source of difference of potential. In an electric circuit there are then two essential parts; the device to convert some form of energy into electrical energy and a device to convert this electrical energy into some other form.

Properties of an Electric Circuit.—If a conductor is joined to the terminals of a source of difference of potential it acquires many new properties which may be classified as physiological, thermal, luminous, electrolytic and magnetic.

Physiological effects are shown by the shock experienced when parts of the body are brought near a highly charged conductor. These effects were studied very early by Galvani and others. They now form the subject-matter of the science of electro-therapeutics. Most luminous effects are indirect and are due to a transformation of heat into light. If some other form of energy is converted into light without an appreciable increase in temperature, the transformation is called luminescence or phosphorescence or fluorescence. Many electrical phenomena are accompanied by luminescence. Thermal effects play an important part in the industrial arts as in lighting, heating, welding, electric furnace, etc. There are many other thermo-electric effects. These effects constitute the science of thermo-electricity. When a chemical compound is made part of the conductor in an electric circuit some chemical change usually takes place. These phenomena are studied under the head of electro-chemistry. The magnetic properties of an electric circuit are those involved in the dynamo, electrical machinery, power transmission, etc. These are the most important to the engineer.

Magnetic Effects.—If the conductor of an electric circuit is placed parallel to a magnetic needle and the circuit closed, the needle will be deflected. See Fig. 39. This experiment is called Oersted's experiment.

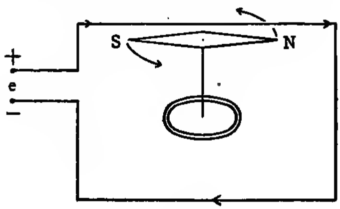


FIG. 39.

It demonstrates the presence of a magnetic field about an electric circuit. Further examination of this magnetic field shows that it has three characteristic properties:

(1st) Starting at any point in this field about the electric circuit and moving always in the direction of the intensity, a closed curve is traced interlinking the circuit. The magnetic field about the conductor is therefore called the "magnetic whorl." In the case of a long straight conductor the magnetic whorl consists of concentric circles. Looking along the conductor from the place of high potential to the place of low potential, the direction of the magnetic whorl is clockwise. See Fig. 40, *b*.

(2d) If the geometrical relations of the separate parts of the circuit remain constant and the circuit as a whole is moved in space, the magnetic field about

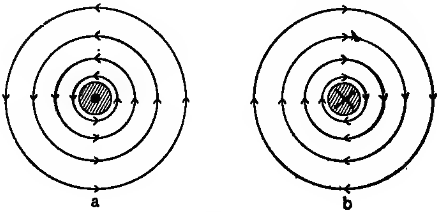


FIG. 40.

the conductor moves with it. If the geometrical relations remain constant and the field at any point is changed in magnitude it will be changed at all other points in the same ratio. The magnitude of intensity at every point is directly proportional to some property of the circuit. The magnitude of the intensity at points equally distant from the conductor approaches the same value as the distances become very small. Therefore the property of the conductor to which the intensity of field is directly proportional is a constant throughout the circuit. This property of the circuit is called "*current.*"

(3d) The magnitude of field due to the current is independent of the permeability, but the magnetic induction depends upon it.

Current.—That property of the electric circuit to which the intensity of magnetic field is directly proportional, is called *current* or *current strength.* By convention its positive direction is said to be from the point of high to the point of low potential.

$$i = k\mathscr{H} \tag{224}$$

In which $\mathscr{H}$ = intensity of the field.
i = current.
k = a constant, the value of which depends upon the geometric arrangement of the circuit, location of the point and the units in which the quantities are measured.

Ampere's Formula.—Ampere's formula determines the value of k in equation (224).

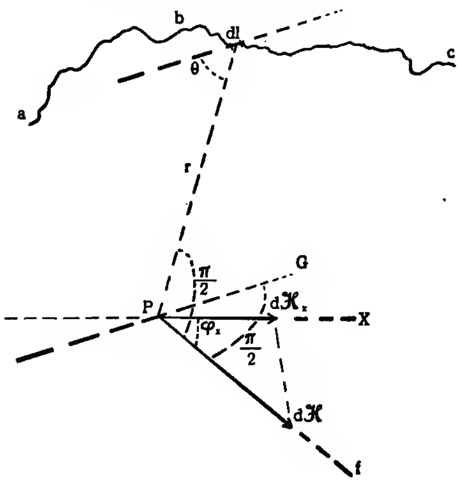


FIG. 41.

Let *abc*, Fig. 41, represent any conductor in space.

dl = any element of length of the conductor.

i = current.

P be any point.

PX represent any direction, through P, taken as X axis.

r = distance from dl to P.

θ = angle between r and dl.

PG represent a line, through P, parallel to dl.

Pf represent a line, through P, perpendicular to r and dl.

$d\mathcal{H}$ = element of field at P due to current in dl.

φ_X = angle between $d\mathcal{H}$ and the X axis.

$d\mathcal{H}_X$ be the X component of $d\mathcal{H}$.

(225) $$\therefore \quad d\mathcal{H}_X = d\mathcal{H}\cdot\cos\varphi_X$$

$d\mathcal{H} \propto i$ By definition.

$d\mathcal{H} \propto dl$ By experiment.

$d\mathcal{H} \propto \frac{1}{r^2}$ By experiment.

$d\mathcal{H} \propto \sin\theta$ By experiment.

$$\therefore \quad d\mathcal{H} \propto \frac{i\cdot\sin\theta\cdot dl}{r^2}$$

(226) $$d\mathcal{H} = k\frac{i\cdot\sin\theta\cdot dl}{r^2}$$

Combine equations (225) and (226).

(227) $$d\mathcal{H}_X = k\frac{i\cdot\sin\theta\cdot\cos\varphi_X\cdot dl}{r^2}$$

(228) $$\mathcal{H}_X = k \int_0^l \frac{i \cdot \sin\theta \cdot \cos\varphi_X \cdot dl}{r^2}$$

The *unit of current* is called the electro-magnetic unit of current if the unit of length is the centimeter, the unit of intensity is the electro-magnetic unit of intensity, and the constant, k, is equal to one. ∴ In electro-magnetic units:

(229) $$\mathcal{H}_X = \int_0^l \frac{i \cdot \sin\theta \cdot \cos\varphi_X \cdot dl}{r^2}$$

An illustration of *unit current*, in the electro-magnetic system, is such a current that, in unit arc of unit circle it produces unit field at the center. The *practical unit of current* is called the *ampere* and is defined as one tenth of the electro-magnetic unit of current.*

One ampere = $(10)^{-1}$ e.m.u. of current.

or

(Number of amperes) $(10)^{-1}$ = (Number of e.m.u. of the same current).

Equations similar to (229) can be written for components along the other axes.

(230) $$\mathcal{H}_Y = \int_0^l \frac{i \cdot \sin\theta \cdot \cos\varphi_Y \cdot dl}{r^2}$$

(231) $$\mathcal{H}_Z = \int_0^l \frac{i \cdot \sin\theta \cdot \cos\varphi_Z \cdot dl}{r^2}$$

(232) $$\mathcal{H} = \sqrt{\mathcal{H}_X^2 + \mathcal{H}_Y^2 + \mathcal{H}_Z^2}$$

* See appendix on International Units.

The direction cosines of $\mathcal{H}$ are:

$$\cos \alpha = \frac{\mathcal{H}_X}{\mathcal{H}} \tag{233}$$

$$\cos \beta = \frac{\mathcal{H}_Y}{\mathcal{H}} \tag{234}$$

$$\cos \gamma = \frac{\mathcal{H}_Z}{\mathcal{H}} \tag{235}$$

Thus the resultant field at any point for any conductor may be completely determined in magnitude and direction. Usually the X axis may be taken in the direction of the resultant field so that equation (229) is sufficient.

Field at the Center of a Circular Conductor.— Let ABC, Fig. 42, be a circular conductor of radius a.

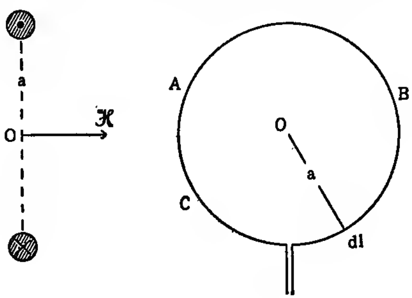


FIG. 42.

Let P be the point at the center. Select as X axis, the line passing through P which is perpendicular to the plane of the coil. Let dl be an element of length of the conductor.

$$\mathcal{H}_X = \int_0^l \frac{i \cdot \sin\theta \cos\varphi_X \cdot dl}{r^2}. \tag{229}$$

For any dl

$$\theta = \frac{\pi}{2}$$

$$\sin\theta = 1$$

$$\cos\varphi_X = 1$$

$$r = a$$

$$l = 2\pi na$$

$$\mathcal{H}_X = \frac{i}{a^2}\int_0^{2\pi na} dl$$

(236) $$\mathcal{H}_X = \frac{2\pi ni}{a}$$

(237) $$i = \frac{a}{2\pi n} \cdot \mathcal{H}_X$$

In which $\mathcal{H}_X$ = intensity at the center, in e.m.u.
a = radius in centimeters.
i = current in e.m.u.
n = number of turns of wire.

If $\mathcal{H}_X$ is compared with $\mathcal{H}_e$, the horizontal component of the earth's field, by the method of the magnetometer, (21), i can be expressed in terms of mass, length, time, and permeability of air taken as one. Such a determination is called an "absolute determination" of current.

Absolute Tangent Galvanometer.—The absolute tangent galvanometer is an instrument for the absolute determination of current, based on the above principle and that of the magnetometer. The instrument consists of a small magnetic needle, suspended by a torsionless fiber, at the center of the coil. The plane of the coil is placed in the plane of the magnetic meridian so that the field, $\mathcal{H}_X$, due to the current is perpendicular to the horizontal component, $\mathcal{H}_e$, of the earth's field.

$$\frac{\mathcal{H}_X}{\mathcal{H}_e} = \tan \theta_i \qquad \text{From (21).}$$

In which θ_i is the angle of deflection. It is observed by means of a telescope and scale. See appendix.

$$(238) \qquad i = \frac{2\pi n}{a} \cdot \mathcal{H}_e \tan \theta_i$$

From (21) and (237).

In which $\mathcal{H}_e$ is determined by the magnetometer.

$n =$ number of turns of wire in the coil.

$$\mathcal{H}_e = \left(\frac{4\pi^2}{T^2} \cdot \frac{I}{d^3 \tan \theta_P}\right)^{\frac{1}{2}} \qquad (32)$$

Thus i can be determined in terms of mass, length, time, and the permeability of air taken as one.

If the instrument is used merely to compare currents:

$$(239) \qquad i = k \tan \theta_i$$

In which

(240)
$$k = \frac{a}{2\pi n} \cdot \mathcal{H}_e$$

provided the conditions are such that $\mathcal{H}_e$ may be considered constant. See paper on Terrestrial Magnetism.

Intensity of Field in the Axis of a Flat Coil. —

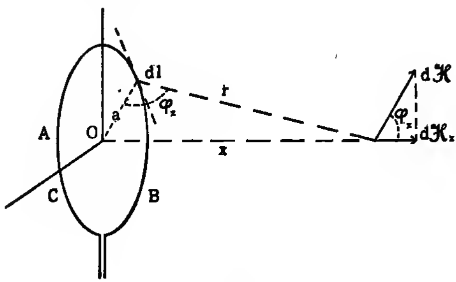


FIG. 43.

Let ABC, Fig. 43, be a flat circular coil of radius a.
Let n = number of turns in the coil.
Let P be a point in the axis of the coil.
Select the X axis along the axis of the coil and the origin at the center, 0.

$$\mathcal{H}_X = \int_0^l \frac{i \cdot \sin\theta \cdot \cos\varphi_X \cdot dl}{r^2} \qquad (229)$$

In this case

$$r = \sqrt{a^2 + x^2}$$

From the figure:

$$\sin\theta = 1$$

$$\cos\varphi_X = \frac{a}{\sqrt{a^2+x^2}}$$

$$l = 2\pi an$$

(241) $$\mathcal{H}_X = \frac{i\cdot a}{(a^2+x^2)^{3/2}}\int_0^{2\pi an} dl$$

$$\int_0^{2\pi an} dl = 2\pi an$$

(242) $$\mathcal{H}_X = \frac{2\pi na^2}{(a^2+x^2)^{3/2}}\cdot i$$

If $x = 0$

(243) $$\mathcal{H}_X = \frac{2\pi n}{a}\cdot i$$

Compare with (236).

Field Due to a Solenoid.—A cylindrical coil or helix the length of which is large compared with its diameter, is called a solenoid.

Let *gh*, Fig. 34, represent a section of such a solenoid.

Let N = total number of turns of wire on the solenoid.

l = length of the solenoid.
a = radius of the solenoid.

Let P be a point at the middle of the axis.

Let dx = element of length of the solenoid.

r = distance from P to dx.

x = projection of r on the axis.

n = number of turns of wire in dx.

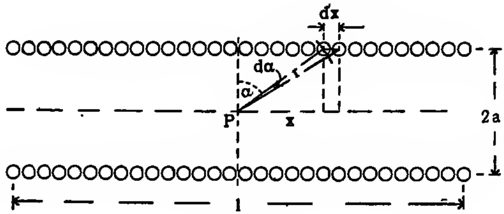


FIG. 44.

$$d\mathcal{H}_X = \frac{2\pi a^2 n}{(a^2 + x^2)^{3/2}} \cdot i \tag{242}$$

$$n = \frac{N}{l} \cdot dx \qquad \text{By definition.}$$

$$d\mathcal{H}_X = \frac{2\pi a^2 N}{(a^2 + x^2)^{3/2} \cdot l} \cdot i \cdot dx$$

From the figure:

$$- r\, d\alpha = dx \cos \alpha$$

$$\cos \alpha = \frac{a}{r}$$

$$r = (a^2 + x^2)^{\frac{1}{2}}$$

$$d\mathcal{H}_X = \frac{2 \cdot \pi \cdot N \cdot i}{l} \cdot \cos \alpha \cdot d\alpha \tag{244}$$

The limiting values of the angle α are 0, when r is perpendicular to the X axis, and some angle, say α_1, when r joins P and the last turn, h. In the latter case:

$$\alpha_1 = \tan^{-1}\left(\frac{l}{2a}\right)$$

For both halves of the solenoid:

$$(245) \qquad \mathcal{H}_x = 2 \cdot \frac{2 \cdot \pi \cdot N \cdot i}{l} \cdot \int_0^{\alpha_1} \cos \alpha \cdot d\alpha$$

From calculus

$$(246) \qquad \int_0^{\alpha_1} \cos \alpha \cdot d\alpha = \sin \alpha \Big]_0^{\alpha_1}$$

$$(247) \qquad \mathcal{H}_x = 4 \cdot \pi \cdot i \cdot \frac{N}{l} \cdot \sin \alpha_1$$

If the length of the solenoid is more than ten times the diameter:

$$\alpha \doteq \frac{\pi}{2}$$

$$\therefore \quad \sin \alpha_1 \doteq 1$$

(See appendix on functions of small angles.)

$$(248) \qquad \mathcal{H}_x = 4 \cdot \pi \cdot \frac{N}{l} \cdot i$$

In which i is measured in e.m.u.

Field Due to a Straight Wire of Infinite Length.— Let AB, Fig. 45, represent any portion of a straight wire of infinite length.

Let P be any point.

d = perpendicular distance from P to the wire.

Let P' be the foot of the perpendicular drawn from P to the wire.

Let dl = any element of length of the wire.

l = distance from P' to dl.

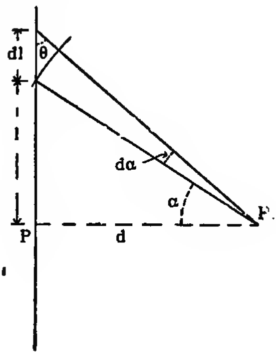


Fig. 45.

The field at P, due to the entire wire, is twice that due to the length from P' to $+\infty$.

$$\mathcal{H}_X = 2\int_0^\infty \frac{i \cdot \sin\theta \cdot \cos\varphi_X \cdot dl}{r^2} \qquad \text{From (229).}$$

Select the X axis through the point P and perpendicular to both d and l. For any dl:

$$\cos\varphi_X = 1$$

$$dl \cdot \sin\theta = r \cdot d\alpha \qquad \text{From figure.}$$

$$r = \frac{d}{\cos\alpha} \qquad \text{From figure.}$$

As l varies from 0 to $+\infty$, α varies from 0 to $\pi/2$.

(249) $$\mathcal{H}_X = 2\frac{i}{d}\int_0^{\pi/2} \cos\alpha \cdot d\alpha$$

From calculus:

$$\int_0^{\pi/2} \cos\alpha \cdot d\alpha = \sin\alpha\Big]_0^{\pi/2}$$

(250) $$\mathcal{H}_X = \frac{2i}{d}$$

This is called Biot and Savart's law. Compare this equation with (136).

Verification of Ampere's Formula.—The above equation, (250), furnishes one of the most rigorous tests of Ampere's formula. Let AB, Fig. 46, represent

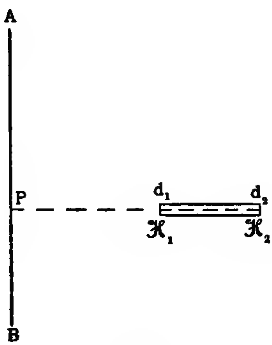


FIG. 46.

a portion of the wire. Arrange a magnet free to rotate about AB as an axis. Let L = moment tending to rotate the magnet about AB as an axis. Let L_1 and L_2 be the moments tending to rotate the north-seeking and the south-seeking poles, respectively, about AB as an axis.

$$L = L_1 + L_2$$

$$L_1 = f_1 d_1$$

$$L_2 = f_2 d_2$$

$$f_1 = m \cdot \mathcal{H}_1$$

$$f_2 = - m \cdot \mathcal{H}_2$$

(251) $$L = m(\mathcal{H}_1 d_1 - \mathcal{H}_2 d_2)$$

But

$$\frac{\mathcal{H}_1}{\mathcal{H}_2} = \frac{d_2}{d_1} \qquad \text{By deduction (250).}$$

(252) $$\therefore \quad L = 0 \qquad \text{By deduction.}$$

Experimentally it is found that there is no measurable tendency to rotate about AB as an axis.

$$\therefore \quad L = 0 \qquad \text{Experimentally.}$$

In a similar way all other deductions made from Ampere's formula can be verified within the limits of experimental error.

PROBLEMS.

61. A magnet pole of strength 25 e.m.u. is placed at the center of a circular coil of diameter 0.3 meter. If a current of 0.5 ampere is passing through the coil, what force will be exerted upon the pole? Ans. 0.523+ dyne.

62. Find the intensity of field at the center of a circular coil and find its component in a direction making an angle of 30 degrees with the axis of the coil. The current in the coil is 10 amperes, diameter of the coil is 40 cm., and there are 5 turns of wire in the coil.

Ans. 1.57+ dynes per unit pole.
1.36 dynes per unit pole.

63. What current must be sent through a tangent galvanometer consisting of 5 turns of wire bent into a circle 45 cm. in diameter, in order that the needle shall be deflected 30 degrees where $\mathcal{H}_e = 0.18$ dyne per unit pole? Ans. 0.743 ampere.

64. Calculate the current which will deflect a tangent galvanometer 45 degrees, if the galvanometer consists of 7 turns of wire, 18 cm. in diameter, set up in a field of horizontal component .198 dyne per unit pole? Ans. .405 + ampere.

65. Find the field strength 16 cm. from the center of a coil in the line of its axis, if the coil carry 0.5 ampere and be 24 cm. in diameter.

Find the force on a pole of 30 e.m.u. if placed at this point.

Ans. .00565 dyne per unit pole.
.17 – dyne.

66. What is the current in a long straight conductor if a force of 45 dynes is exerted upon a pole of 75 e.m.u. placed at a distance of 30 cm. from the wire? Ans. 90 amperes.

67. It is desired that a field of 5 dynes per unit pole be produced at the middle point of the axis of a solenoid. If the coil is 20 cm. long and has 250 turns, how many amperes of current are necessary to produce the field? Ans. 0.318 ampere

CHAPTER VIII.

QUANTITY.

Quantity.—In the electro-magnetic system, *quantity of electricity* is defined as the product of the current into the time.

(300) $$q = i\, t$$

In which i = the current.

t = the interval of time. It is taken so small that i may be considered as constant.

q = quantity of electricity.

Unit of Quantity.—The *unit of quantity* is called the electro-magnetic unit of quantity, if the unit of current is the e.m.u. of current and the unit of time is the second. The unit of quantity is called the "*coulomb*" or *practical unit of quantity*, if the unit of current is the ampere and the unit of time is the second. Thus the coulomb is one tenth of an electro-magnetic unit of quantity.

One coulomb = $(10)^{-1}$ e.m.u. of quantity.

(The number of coulombs) $(10)^{-1}$ = (Number of e.m.u. of quantity).

Electrolysis.—*Measurement of Quantity.* If a chemical compound forms part of the conductor joined to a source of difference of potential, chemical decomposition usually takes place. The compound decomposed is called an "*electrolyte.*" The solid conductors in contact with it are called the "*electrodes.*" One of the electrodes is called the "*anode*" and the other is called the "*cathode.*" The positive direction of the current in the electrolyte is from anode to cathode. This combination of electrolyte, electrodes, and containing vessel, is called an "*electrolytic cell.*" The products of decomposition are set free from the electrolyte at the electrodes. They may or may not combine with the electrodes, depending upon their chemical nature.

Let Fig. 47 represent an electric circuit containing several electrolytic cells in series.. If the circuit is

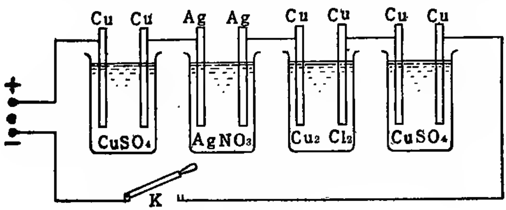


FIG. 47.

closed by closing the switch, K, the time, t, during which decomposition takes place, will be the same for each cell. The products of decomposition set free at the electrodes of the several cells, are found to be directly proportional to their combining weights. It is also found that the amount of the substance set free at each electrode, is directly proportional to the product, $i \cdot t$, or quantity of electricity.

These two important facts were first established by Faraday. For this reason the following laws are known as Faraday's laws of electrolysis:

First. The masses of the substances liberated at the electrodes are proportional to their combining weights.

Second. The mass of an electrolyte decomposed by an electric current, is proportional to the quantity of electricity, q.

(301) $$m_2 - m_1 = c \cdot i \cdot t$$

In which m_1 = mass of the cathode before closing the circuit.

m_2 = mass of the cathode after the time t.

c = a constant depending on the nature of the electrolyte. It is called the *electro-chemical equivalent* of the element.

Coulometer.—A *coulometer* is an instrument for the determination of quantity of electricity, based upon the above principle (301). A common form is that known as the copper coulometer. It consists of two copper electrodes, and dilute copper sulphate solution as the electrolyte. If the coulometer and tangent galvanometer are joined in series to a constant source of difference of potential (Fig. 48) and current and time noted, the electrochemical equivalent of the copper, or the coulometer constant, may be determined in terms of mass, length, time, and permeability of air taken as one.

For the copper sulphate coulometer:

$$c = 0.000329 \text{ grams per coulomb}$$

(302) $q = \dfrac{m_2 - m_1}{.000329}$ for copper in $CuSO_2$ solution

(303) $q = \dfrac{m_2 - m_1}{.001118}$ for silver in $AgNO_3$ solution

In general

(304) $q = 96530 \dfrac{m_2 - m_1}{\text{combining weight}}$

Approximately 96,530 coulombs liberate a mass of the

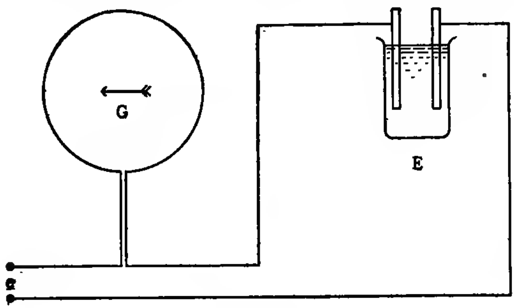


FIG. 48.

radical, numerically equal (in the number of grams) to its combining weight.

PROBLEMS.

68. Find the time necessary for 0.06 amperes to deposit 0.5 gram of silver from a silver nitrate solution.

Ans. 2 hrs. 4 min. and 10 sec.

69. Calculate the current that deposits 0.315 gram of copper from a copper sulphate solution, in 30 minutes.

Ans. 0.532 ampere.

70. The weight of a silver plate cathode was 30.3726 g. before the deposit on it and 33.0701 g. after deposition which lasted half an hour. Find the average current in amperes.

Ans. 1.34+ amperes.

71. A current of 0.05 ampere was passed through a solution of nickel sulphate for one hour. The amount of nickel deposited at the cathode was 0.055 gram. Find the electro-chemical equivalent of nickel.

Ans. .000305 gram per coulomb.

72. A tangent galvanometer is connected as shown in Fig. 48. to an electrolytic cell containing silver nitrate solution. The anode is of silver and cathode of platinum. The galvanometer coil is 30 cm. in diameter and is set up at a point where the horizontal component of the earth's field is 0.20 dyne per unit pole, On closing the circuit the deflection of the needle was observed and an average found to be 3 degrees. What length of time must the deposit take place in order that .2236 gram of silver shall be deposited? Ans. 800 sec.

73. A tangent galvanometer of 28 turns, having a diameter of 22 cm., is set up where the horizontal intensity of the earth's field is 0.2 dyne per unit pole. A current which deflects the needle 45 degrees will take how long to deposit .2 gram of Cu on the cathode of a coulometer, if the electrochemical equivalent of Cu is .000329 gram per coulomb? Ans. 1 hr. 21 min. 3 sec.

CHAPTER IX.

RESISTANCE, DIFFERENCE OF POTENTIAL AND CAPACITY.

Resistance.—When a conductor is joined to a source of difference of potential, heat is generated in the conductor. Experimentally it has been found that the quantity of heat developed is directly proportional to the square of the current and also directly proportional to the time.

$$W_H \propto i^2$$

$$W_H \propto t$$

$$W_H = Ri^2t \tag{305}$$

This expression is called Joule's law. R is the constant of proportionality between the work and i^2t. This constant is called the *resistance* of the conductor in which the heat, W_H, is developed.

Unit of Resistance.—The *unit of resistance* is called the electro-magnetic unit of resistance if the unit of work is the erg, the unit of time the second, and the unit of current is the electro-magnetic unit of current. The unit of resistance is called the "*ohm,*" or practical unit of resistance if the unit of work is the joule, the unit of time is the second, and the unit of current is the ampere.

$$R = \frac{W_H}{i^2t} \tag{305}$$

If the unit of work is the joule or $(10)^7$ ergs:

$$\frac{10^7}{10^{-2}} = 10^9$$

$\therefore$ One ohm $= 10^9$ e.m.u. of resistance.

$\therefore$ (Number of ohms)$(10)^9 =$ (Number of e.m.u. of resistance)

Verification of Joule's Law.—If part of the conductor joining a source of difference of potential, passes through the inner cup of a calorimeter (*a*, Fig. 49), the heat developed can be measured.

Let $c =$ calorimeter constant.
$M =$ mass of water in the calorimeter.
$\theta =$ the increase in temperature.
$W_c =$ number of calories of heat developed.

(306) $$W_c = (M + c)\theta$$

(307) $$W_H = W_c(4.2)(10)^7$$

(308) $$\therefore \quad W_H = (M + c)\theta(4.2)(10)^7$$

This equation enables us to determine the amount of heat developed, in ergs. If a galvanometer is connected in series with this conductor, *G*, Fig. 49, the current can be measured in e.m.u. and the value of the constant, *R*, can be determined in e.m.u. This is called an absolute determination of resistance.

Standard Resistances.—Convenient standard resistances consist of coils of wire carefully adjusted and connected to heavy metallic terminals.

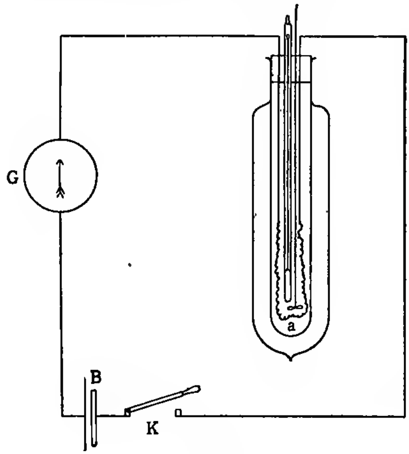


FIG. 49.

Resistance Boxes.—For less accurate comparisons resistance boxes are used. In these a number of resistance coils are arranged so as to permit the use of resistances having a wide range of values, varying in uniform steps.

Difference of Potential.—Difference of potential has been defined in the electrostatic system as:

$$e = \frac{w}{q} \qquad \text{From (112).}$$

It is defined by the same relation in the electromagnetic system.

(309) $$e = \frac{w}{q}$$

The *unit of difference* of potential is called the electromagnetic unit of difference of potential, if the unit of work is the erg and the unit of quantity is the electromagnetic unit of quantity. The unit of difference of potential is called the volt, or practical unit of difference of potential, if the unit of work is the joule and the unit of quantity is the coulomb. Since the joule is 10^7 ergs and the coulomb is 10^{-1} e.m.u.

$\therefore$ One volt $= 10^8$ e.m.u. of potential difference.

$\therefore$ (Number of volts)$(10)^8$ = (Number of e.m.u. of potential difference)

Ohm's Law.—The relation of difference of potential to resistance can be derived from equations (300), (305), and (309). For values of t so small that i may be considered as constant:

$$W_H = Ri^2t \qquad (305)$$

(310) $$\frac{W_H}{q} = Ri$$ From (300) and (305).

(311) $$e = Ri$$ From (310) and (309) or (112).

This relation is called Ohm's law. It may be used for an absolute determination of difference of potential, provided the current and resistance are stated in terms of electromagnetic units.

Ratio of Units.—If a plate condenser of known dimensions is joined to a source of difference of potential which is measured by means of an electrometer,

$$q_e = e_e C_e \tag{200}$$

$$C_e = \frac{\kappa A}{4\pi d} \tag{205}$$

$$e_e = \sqrt{\frac{8\pi d_e^2}{A_e} \cdot f} \tag{154}$$

$$q_e = \frac{\kappa A}{4\pi d} \cdot \sqrt{\frac{8\pi d_e^2}{A_e} \cdot f} \text{ in e.s.u.} \tag{312}$$

If the condenser is disconnected from the source and connected in series with the silver coulometer we have:

$$q_m = \frac{m_2 - m_1}{.01118} \text{ in e.m.u.} \tag{313}$$

Assuming that the quantity of electricity measured by the coulometer is the same as that discharged by the condenser, it is found that one e.m.u. of quantity is equal to $3(10)^{10}$ e.s.u., of quantity within the limits of experimental error.

$$e_e q_e = e_m q_m \tag{314}$$

since the unit of work in the two systems is the same.

∴ The electrostatic unit of difference of potential is equal to $3(10)^{10}$ electro-magnetic units of difference of potential. Since the volt is equal to $(10)^8$ e.m.u., one e.s.u. of difference of potential is equal to 300 volts.

The assumption made in the preceding paragraph may be verified as follows: If in Fig. 49 the difference of potential across the measured resistance, R, is calculated in e.m.u., and measured by an electrometer in e.s.u., it is found that one e.s.u. of potential difference is equal to $3(10)^{10}$ e.m.u. of potential difference. In a similar way all other deductions made from this assumption may be verified within the limits of experimental error.

Primary Cells.—If in an electrolyte, there be immersed two electrodes, for one of which the electrolyte has a greater affinity than for the other, there will be a difference of potential between the electrodes. Such an arrangement is called a *primary cell.* If the electrodes are connected by a conductor, current will be produced in the circuit. The chemical energy of the cell is converted into electrical energy; and the electrical energy is converted, by the conductor, into heat or other forms. The work per unit quantity, in converting the chemical energy into electrical energy, is called the electro-motive force (e.m.f.), of the cell.

Standard Cells.—A standard cell is a primary cell which maintains a definite constant difference of potential between the terminals of the cell. To maintain the difference of potential constant, the cell is used as a source of minute currents only. Some common forms of standard cells are the following:

Cell.	Diff. of Pot.	Temp.
Clark	1.433 volt.	15° C.
Weston	1.0183 volt.	20° C.

Capacity.—Capacity has been defined in the electrostatic system as:

$$(315) \qquad C = \frac{q}{e} \qquad (200)$$

It is defined by the same relation in the electro-magnetic system. The *unit of capacity* is called the electro-magnetic unit of capacity if the quantity and the difference of potential are expressed in electro-magnetic units. As an illustration, unit capacity, in the electro-magnetic system, is a capacity of such magnitude that it would be charged to one electro-magnetic unit of difference of potential by one e.m.u. of quantity of electricity. Since the e.m.u. of quantity is equal to $3(10)^{10}$ e.s.u., and the e.m.u. of difference of potential is equal to $\frac{1}{3(10)^{10}}$ e.s.u., the e.m.u. of capacity is $9(10)^{20}$ e.s.u. The *unit of capacity* is called the *farad*, or practical unit of capacity, if the difference of potential is expressed in volts and the quantity is expressed in coulombs; *e. g.*, the farad is a capacity of such magnitude that it would be charged to a difference of potential of one volt by one coulomb of electricity. Since the coulomb is $(10)^{-1}$ e.m.u., and the volt is $(10)^8$ e.m.u., the farad is $(10)^{-9}$ e.m.u. of capacity. This is a very large unit. The one millionth part of the farad, called the micro-farad, is commonly used.

One e.m.u. of capacity $= 9(10)^{20}$ e.s.u. of capacity

$\therefore$ (Number of e.m.u. of capacity) $\cdot (9) \cdot (10)^{20}$

$=$ (Number of e.s.u. of capacity)

One farad $= (10)^{-9}$ e.m.u. of capacity

$\therefore$ (Number of farads)

$= (10)^9 \cdot$ (Number of e.m.u. of capacity)

One micro-farad $= (10)^{-6}$ of a farad.

$\therefore$ (Number of micro-farads) $=$

$(10)^6 \cdot$ (Number of farads)

PROBLEMS.

74. What current would have to flow for an hour through a resistance of 42 ohms in order that the heat produced might raise the temperature of one kilogram of water from 0 to the boiling point? Ans. 1.66 amp.

75. The resistance of a hot carbon filament in an incandescent lamp was 200 ohms. If the filament is connected, by leads of negligible resistance, to a difference of potential of 110 volts, how much electrical energy is converted in one hour?

Ans. 217,800 joules.

76. A parallel plate condenser is made of 101 rectangular plates each 7.43 cm. $\times$ 19 cm. The intermediate layers are of paraffine 0.005 cm. thick and of specific inductive capacity twice that of air. What is its capacity in micro-farads?

Ans. 0.499 micro-farad.

77. If the plates of a condenser, having a capacity of 0.7 micro-farad, are connected to a difference of potential of 100 volts, what quantity of electricity will be on the plates?

Ans. .00007 coulomb.

78. If a static machine is made to discharge through a circuit containing a silver coulometer, how many e.s.u. of electricity are necessary to deposit one milligram of silver?

Ans. $(2.7)(10)^9$ e.s.u. of quantity.

CHAPTER X.

NETWORKS OF CONDUCTORS.

Kirchhoff's Laws.—If the conductor joining a source of difference of potential, consists of several parts, as in Fig. 50 or Fig. 51 or any combination of the two, it is called a network of conductors. If at any point in a network of conductors, the currents whose positive directions are toward the point are reckoned positive and the currents whose positive directions are away from the point are reckoned negative, the algebraic sum of the currents is equal to zero.

$$\Sigma i = 0 \tag{320}$$

This law was experimentally demonstrated by Kirchhoff.

If, starting at any point in a network of conductors, a quantity of electricity makes a complete cycle back to its initial position, the work done upon it is equal to the work done by it. The amount of energy converted into electrical energy, in going from places of low to those of high potential, must equal the amount of electrical energy converted into heat in going from points of high to points of low potential.

$$\Sigma(\text{e.m.f.}) - \Sigma Ri = 0$$

or if the fall of potential due to resistance is reckoned as a negative electro-motive force, the expression may be written:

(321) $$\Sigma e = 0$$

This is called Kirchhoff's law for electro-motive forces. These two laws, together with Ohm's law, are used to establish many propositions concerning the electric circuit.

Resistances in Series.—If a number of resistances R_1, R_2, R_3, etc., are joined to a difference of potential, e, as in Fig. 50, they are said to be joined in series.

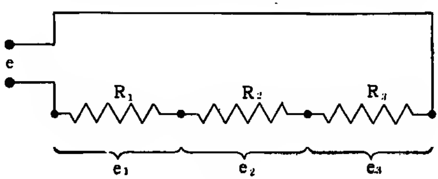


FIG. 50.

Let R = resistance of the combination.

e = difference of potential at the ends of the resistance, R.

i = current in the resistance, R.

$$e = Ri \qquad (311)$$

Let e_1 = potential difference between the ends of R_1.

i_1 = current in R_1.

etc.

$$e_1 = R_1 i_1 \qquad \text{From (311).}$$
$$e_2 = R_2 i_2 \qquad \text{From (311).}$$
$$e_3 = R_3 i_3 \qquad \text{From (311).}$$
etc.

(322) $$e = e_1 + e_2 + e_3 + \cdots \qquad (321)$$
$$i = i_1 = i_2 = i_3 = \text{etc.}$$

Substituting in equation (322):

(323) $$Ri = R_1 i_1 + R_2 i_2 + R_3 i_3 + \cdots$$

Divide by i.

(324) $$R = R_1 + R_2 + R_3 + \cdots$$

If

$$R_1 = R_2 = R_3 = \cdots$$

(325) $$R = nR_1$$

Resistances in Parallel.—If a number of resistances, R_1, R_2, R_3, etc., are joined to a source of difference of potential, e, as in Fig. 51, they are said to be joined in parallel.

Let R = resistance of the combination.
e = difference of potential at the ends of R.
i = current in R.

$$e = Ri \qquad (311)$$

Let e_1 = potential difference between the ends of R_1.
i_1 = current in R_1.
etc.

$$e_1 = R_1 i_1 \qquad \text{From (311).}$$

$$e_2 = R_2 i_2 \qquad \text{From (311).}$$

$$e_3 = R_3 i_3 \qquad \text{From (311).}$$

etc.

(326) $$i = i_1 + i_2 + i_3 + \cdots \qquad (320)$$

$$e = e_1 = e_2 = e_3 = \cdots, \text{ etc.}$$

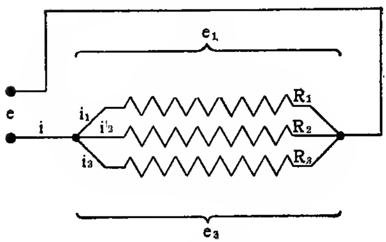


FIG. 51.

Substituting in equation (326):

(327) $$\frac{e}{R} = \frac{e_1}{R_1} + \frac{e_2}{R_2} + \frac{e_3}{R_3} + \cdots$$

divide by e.

(328) $$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots$$

If

$$R_1 = R_2 = R_3 = \cdots$$

(329) $$R = \frac{1}{n} R_1 \quad \text{Compare with (325).}$$

In which n = number of conductors joined in parallel. Compare these equations, (324) and (328), with those for capacity in series and capacity in parallel, (206) and (208).

Specific Resistance. — In homogeneous conductors of uniform cross section:

(339) $$R \propto l$$ (325)

In which l is the length of the resistance, R.

If two homogeneous conductors of uniform cross-section and equal in length, are joined to the same source of difference of potential, points, in the two conductors, which are equally distant from the same terminal, will be at the same potential. ∴ The conductors may be placed together in continuous contact, without changing electrical conditions. A conductor of uniform cross-section may be regarded as made up of n conductors in parallel. ∴ For a conductor of uniform cross-section, s,

(331) $$R \propto \frac{1}{s}$$ (329)

Combining these two expressions, (330) and (331),

(332) $$R = \gamma \cdot \frac{l}{s}$$

In which γ is the constant of proportionality. It is called *specific resistance*. It is numerically equal to the resistance, in ohms, of a conductor one cm. long and one sq. cm. in cross-section. Values for some substances are given in the following table.

Substance.	Specific Resistance at 0° C.	
Silver (annealed)	.00000146	ohm, for a conductor one cm. long and one sq. cm. in cross-section.
Silver (hard drawn)	158	
Copper (annealed)	158	
Copper (hard drawn)	162	
Aluminium (annealed)	291	
Platinum (annealed)	904	
Iron (annealed)	969	
Nickel (annealed)	.00001243	
Mercury	941	
German silver	209	
German silver (commercial wire)	35	
Carbon filament	.004	

Temperature Coefficient.—The resistance of a conductor is found, by experiment, to depend upon the temperature. It may be expressed by the empirical formula:

$$R = R_0(1 + \alpha T + \beta T^2 + \cdots) \tag{333}$$

In which T = change in temperature. If T is small terms with powers of T higher than the first may be neglected. If both β and T^2 are very small.

$$\therefore \quad R = R_0(1 + \alpha T) \tag{334}$$

The coefficient α is called the "*temperature coefficient.*" For small differences of temperature the temperature coefficient may be considered a linear function of the change in temperature. For pure metals, $\alpha \doteq .00366$. For other substances α may be positive, negative, or equal to zero. The value for carbon filament is negative. The following table gives the mean value of the temperature coefficient between 100° C. and − 100° C.

Substance.	Mean. Temp. Coeff. between — 100° and + 100° C.
Aluminium substance (hard drawn)	.00446
Copper (electrolytic, annealed)	.00431
Platinum (annealed)	.00341
Silver (pure wire)	.00377
German silver (commercial wire)	.00035
Phosphor-bronze (commercial wire)	.00070
Carbon filament	−.0003 ±
Platinoid (Martino's platinoid) (1 to 2 per cent. of tungsten)	.00025
Platinum silver (66.7 per cent. silver + 33.3 per cent. platinum)	.00024
Manganin (Cu 84 per cent., Mn 12 per cent., Ni 4 per cent.)	.0000 at 15°

In the construction of standard resistances it is preferable to use substances having a very small temperature coefficient. For this purpose various alloys have been placed on the market. They are known to the "trade" by special names, *e. g.*, manganin, constantan, etc. Manganin has, at ordinary temperatures, a very small coefficient which is positive at ordinary temperatures, approaches zero as the temperature is increased and finally becomes negative.

Wheatstone's Bridge.—A common method for comparison of resistances, is that known as the Wheatstone-bridge method. The bridge consists essentially of four arms, R_1, R_2, R_3, R_4, which may be conventionally arranged as in Fig. 52. In the figure, e is a source of difference of potential and G is the galvanometer. When there is no deflection of the galvanometer,

$$\dot{i}_G = 0$$

$$\therefore \quad V_2 = V_3$$

also

$$\dot{i}_1 = \dot{i}_2 \qquad \text{From (320).}$$

and

$$\dot{i}_3 = \dot{i}_4 \qquad \text{From (320).}$$

$$e_1 = V_2 - V_1$$

$$e_3 = V_3 - V_1$$

$$\therefore \quad e_1 = e_3$$

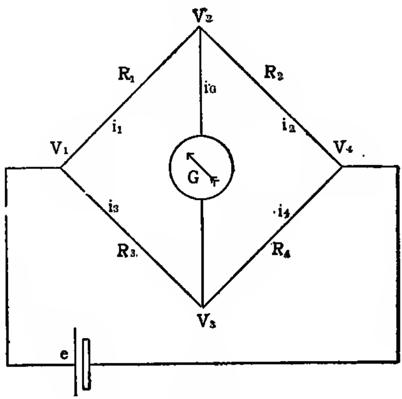


FIG. 52.

In a similar way:

(335) $$e_2 = e_4$$

By Ohm's law

(336) $$e_1 = R_1 i_1$$

(337) $$e_2 = R_2 i_2$$

(338) $$e_3 = R_3 i_3$$

(339) $$e_2 = R_4 i_4$$

Dividing (336) by (337) and substituting for i_1:

(340) $$\frac{R_1}{R_2} = \frac{e_1}{e_2}$$

Dividing (338) by (339) and substituting for i_3:

(341) $$\frac{R_3}{R_4} = \frac{e_3}{e_4}$$

But

$$\frac{e_1}{e_2} = \frac{e_3}{e_4}$$

(342) $$\therefore \quad \frac{R_1}{R_2} = \frac{R_3}{R_4}$$

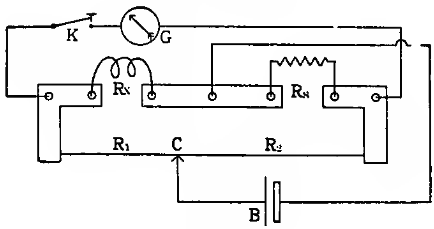


Fig. 53.

Slide-wire Bridge.—In the slide-wire bridge or sliding contact bridge, two of the arms, R_1 and R_2, Fig. 53, consist of a homogeneous wire of uniform cross-section. The sliding contact, C, is adjusted until a point on the wire is found where, on pressing both keys, C and K, no current is indicated by the galvanometer, G. A comparison of this bridge with that shown in Fig. 52 will show that:

(343) $$\frac{R_1}{R_x} = \frac{R_2}{R_s}$$ From (342).

(344) $$\frac{R_1}{R_2} = \frac{R_x}{R_s}$$ From (343).

Let l_1 = length of the resistance R_1.

l_2 = length of the resistance R_2.

(345) $$\frac{R_1}{R_2} = \frac{l_1}{l_2}$$ From (332).

(346) $$R_x = \frac{l_1}{l_2} R_s$$ From (344) and (345).

Potentiometer.—The potentiometer is an instrument for comparing two differences of potential. A common form, Fig. 54, consists of a wire of uniform cross-section, joined to a constant source of difference of potential, e, which is greater than the difference of potential of either of the two to be compared.

Let e_s and e_x be the standard and the unknown difference of potential to be compared. Throw the switch, K, so as to connect the unknown in the galvanometer circuit. Adjust the sliding contact until a

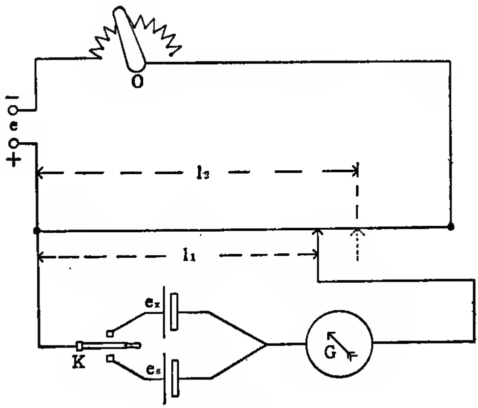


Fig. 54.

length, l_1, is found at which no deflection of the galvanometer is produced. Repeat the process with the standard in the galvanometer circuit. Call the length found l_2. Since the wire is uniform:

(347) $$\frac{l_1}{l_2} = \frac{R_1}{R_2}$$ From (332.)

With the unknown

(348) $$e_1 = R_1 i_1$$

With the standard

(349) $$e_2 = R_2 i_2$$

In which R_1 and R_2 are the resistances of l_1 and l_2.
e_1 and e_2 are the difference of potentials between the ends of l_1 and l_2 respectively.

If the current is kept constant through the wire while the measurements are made:

(359) $$i_1 = i_2$$

(351) $$\frac{e_1}{e_2} = \frac{R_1}{R_2}$$

From (348), (349) and (350).

(352) $$\frac{e_1}{e_2} = \frac{l_1}{l_2}$$ From (351) and (347).

The difference of potential, resistance and current in the galvanometer circuit will be:

(353) $$e_G = R_G i_G$$

(354) $$e_G = e_X - e_1$$

(355) $$e_X - e_1 = R_G i_G$$

But

$$i_G = 0$$ For no deflection.

(356) $$e_X = e_1$$

In a similar way it can be shown that:

(357) $$e_S = e_2$$

(358) $$\frac{e_X}{e_S} = \frac{l_1}{l_2}$$

From (352), (356) and (357).

Power.—In every conversion of energy the power at any instant is defined as the ratio of the work, done in a time t, to the time, provided the time is so small that the ratio may be considered a constant.

(359) $$p = \frac{W}{t}$$ By definition.

If t is not small as indicated above, p represents the average power.

$$W = eq \qquad (309)$$

(360) $$p = \frac{eq}{t}$$

$$q = i \cdot t \qquad (300)$$

(361) $$\therefore \quad p = e \cdot i$$

Which means that, in an electric circuit, the power converted by any part, is equal to the product of the current in that part times the difference of potential across its terminals. The power, p, is in practical units or watts, if the work is in joules and the time is in seconds. Instead of stating the work in joules, electricians frequently state it in "watt-hours," or in "kilowatt-hours."

One watt-hour = 3,600 joules

Equation (361) is the general expression for power consumed between any two points in a circuit. It applies to all forms of electric converters. In the case of a simple conductor the difference of potential between two points is:

$$e = Ri \qquad \text{By Ohm's law.}$$

(362) $$\therefore \; p = Ri^2 \qquad \text{From (361).}$$

These two expressions apply to the resistance between two points within an electrolyte.

Line Loss.—The power consumed in the leads, from the source of potential difference to the point at which it is converted, is sometimes called the "loss in the line" or the Ri^2 loss. The difference of potential across the terminals of the converter is less than that of the source. This difference is called the "drop" in potential difference of the line or the "*Ri* drop."

PROBLEMS.

79. If the addition of 5 ohms to a circuit reduces the current to 0.7 of its former value, how many ohms should be added to reduce the current to 1/4 of its original value, the potential difference remaining constant? What is the resistance of the original circuit?

Ans. 11.67 − ohms, original resistance.
35.0 + ohms, to be added.

80. Two wires have resistances of 36 ohms and 45 ohms respectively. They are connected in parallel so that the total current, in both branches, is 9 amperes. What is the joint resistance, and what is the current in each branch?

Ans. 20 ohms joint resistance.
5 amperes in 36 ohms.
4 amperes in 45 ohms.

81. Six conductors have the following resistance: 10, 15, 16, 20, 24, and 30 ohms respectively. They are connected to an e.m.f. of 15 volts, in the following order: the first two are in parallel, the third in series, and the last three in parallel. What is the current in the first two?

Ans. 0.3 ampere in 10 ohms.
0.2 ampere in 15 ohms.

82. The poles of a battery of 5 cells, in series, are connected by a wire 8 meters long, having a resistance of 1/2 ohm per meter. Each cell has an e.m.f. of 1.4 volts and a resistance of 2 ohms. Find the distance between two points on the wire so that their difference of potential shall be one volt.

How many calories of heat per minute are given off by the wire between these two points?

Ans. 4 meters.
7.16 calories per minute.

83. A length of uniform wire, of resistance 12 ohms, is bent into a circle. Two points, a quarter of the circumference apart, are connected, by a wire of resistance 0.6 ohm, with a battery of resistance 0.5 ohm and e.m.f. of 3 volts. Find the current in the different parts of the circuit.

Ans. 0.895 ampere in 0.6 ohm.
0.224 ampere three-quarters circumference.
0.672 ampere one-quarter circumference.

84. From the specific resistance of copper calculate the resistance of a double line of copper wire, 6.25 kilometers long, and 0.7 cm. in diameter, allowing 2.5 per cent. for "sag" and waste,
Ans. 5.42 ohms.

85. The resistance of a platinum coil at +20° C. is found to be 8.75 ohms. The coil is plunged into a cold expanded gas and its resistance is found to be 5.30 ohms. What is the temperature of the gas? Ans. −96° C.

86. It is found that there is no current through the galvanometer of a "slide-wire" bridge, when contact is made at a point 53.8 cm. from the end of the wire, which is 100 cm. long. If the resistance in the homologous arm of the bridge is 38 ohms, what is the other resistance? Ans. 32.6+ ohms.

87. The ends of an unknown resistance are connected to resistance of 15 ohms and 50 ohms respectively. To the free ends of the two latter resistances, a resistance of 10 ohms is connected. An e.m.f. of 1.2 volts is connected to the point, joining the 10 ohm and the 15 ohm coils, and to the point joining the 50 ohm coil with the unknown. A galvanometer is connected to the point joining the 10 and the 50 ohm coils, and to the point joining the 15 ohm coil to the unknown. When there is no current through the galvanometer find:

(1) Resistance X.
(2) Current in the network.
(3) Current in each resistance.
(4) Potential difference across each resistance.

Ans. (1) 75 ohms.
(2) 0.0333+ ampere.
(3) 0.0133+ ampere in the 15 ohm coil.
0.02 ampere in the 10 ohm coil.
(4) 0.2 volt across the 10 ohm coil.
1.0 volt across the 50 ohm coil.
0.2 volt across the 15 ohm coil.
1.0 volt across the X ohm coil.

88. In comparing the e.m.f. of a dry cell with that of a standard Daniell cell of 1.10 volts, by a wire potentiometer the drop of potential in 53.1 cm. of wire was found to balance the standard. The length required for the dry cell was 69.6 cm. Find its e.m.f.

Ans. 1.44+ volts.

89. How long can a 55 watt incandescent lamp be burned for one dollar if the cost is ten cents per kilowatt-hour?

Ans. 182− hours.

90. 33 kilowatts are required at a certain point. The resistance of the line from the dynamo to that place is 0.00733 ohm. If the difference of potential at that point is 1,000 volts, calculate the power lost in the line. Ans. 8 watts.

91. If the resistance of one heating coil in a car is 500 ohms, and four such coils are joined in parallel, what is the resistance of the heating circuit?

What current flows if a difference of potential of 550 volts is maintained between the ends? Calculate the power in watts. Find the cost of heating the car one hour at $.09 per kilowatt-hour.

Ans. 125 ohms.
4.4 amperes.
2.41+ kilowatts.
$0.22−.

92. If 500 incandescent lamps, in parallel, are supplied with 1/2 ampere each at 100 volts at the lamp terminal, and if the resistance of the line is 0.02 ohm, what is: the total current, power used in the lamps, power lost in the line, and voltage at the dynamo?

Ans. 250 amperes.
25 kilowatts.
1.25 kilowatts.
105 volts.

93. It is desired to transmit 50 kilowatts at 200 volts, a distance of 1 mile, with a loss of 10 per cent. of the power generated. Calculate the resistance of the double line and diameter of the wire.

Ans. 0.08 ohm.
2.86+ cm. diameter.

94. A power station must transmit 500 kilowatts at a 10 per cent. loss, over a double line of copper wire 10 miles long. Compare the resistance of the line and diameter of wire necessary at 100 volts with the resistance and diameter necessary for transmission at 50,000 volts.

Ans. 0.002 ohm and 57.5 cm. diameter.
500 ohms and 0.115 cm. diameter.

CHAPTER XI.

ELECTRODYNAMICS.

Force Exerted on a Conductor Placed in a Magnetic Field.—The magnetic field due to a current is expressed by Ampere's formula:

$$\mathcal{H}_X = \int_0^l \frac{i \sin \theta \cos \varphi_X \, dl}{r^2} \tag{229}$$

If a pole m is placed at the point P, the X component of the force exerted upon the pole is:

$$f_X = m \cdot \mathcal{H}_X \qquad \text{From (2).}$$

(400) $$f_X = \int \frac{m}{r^2} i \cdot \sin \theta \cdot \cos \varphi_X \cdot dl$$

From (229) and (2).

By Newton's third law this is numerically equal to the reaction on the conductor.

$$\beta_m = \frac{m}{r^2} \tag{35}$$

(401) $$f_X = \int \beta_m \cdot i \cdot \sin \theta \cdot \cos \varphi_X \cdot dl$$

In which β_m = the induction of the magnetic field at the element dl. It has the same direction as r.

θ = angle between dl and β_m.
φ_X = angle between the X axis and the perpendicular to the plane of dl and β_m.
f_X = component of the force in the direction of the X axis.

Straight Conductor in a Uniform Field.—If the conductor is straight and the field uniform, β_m, $\sin\theta$, and $\cos\varphi_X$ are constant.

$$f_X = \beta_m \cdot i \cdot \sin\theta \cdot \cos\varphi_X \int_0^l dl \tag{402}$$

$$f_X = l \cdot \beta_m \cdot i \cdot \sin\theta \cdot \cos\varphi_X \tag{403}$$

If the conductor, field and the X axis are mutually perpendicular to each other,

$$\sin\theta = 1$$
$$\cos\varphi_X = 1$$
$$f = l \cdot \beta \cdot i \quad \text{Dropping subscripts.} \tag{404}$$

The force exerted upon a straight conductor perpendicular to a uniform field, is equal to the continued product of the length of the conductor, the current and

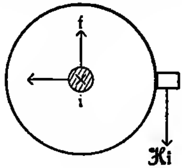


FIG. 55.

the induction. If the length is in centimeters, the current and induction in e.m.u. the force is in dynes. The relation of the current, the field, and the force are shown by diagram in Fig. 55. These directions are those of the right handed system of coordinates.

Parallel Currents.—Let A and B, Fig. 56, represent two parallel currents in the same direction. First

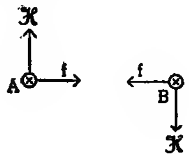


FIG. 56.

consider the field at B due to A. The force on B, according to the relation given above, will be toward A. Similarly, the force on A in the field due to B, will be toward B. ∴ Parallel currents having the same direction attract each other.

Let A and C, Fig. 57, be two parallel currents in

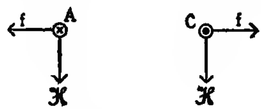


FIG. 57.

opposite directions. The force on C in the field due to A will be away from A, according to the relation of the current, field and force. Similarly the reaction on A will be equal and opposite, *i. e.*, away from C. ∴ Parallel currents in opposite directions repel each other.

Rectangular Coil.—Let $cec'e'$, Fig. 58, represent a rectangular coil free to rotate about an axis parallel to

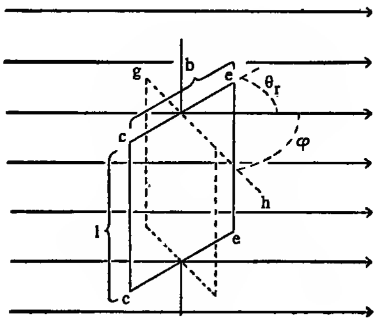


FIG. 58.

one side, and perpendicular to a uniform magnetic field. Since the direction of the forces, exerted upon the end branches, b, are parallel to the axis of rotation, the moment due to them, about this axis is equal to zero.

Let L = sum of the moments tending to produce rotation about the axis, 0, Fig. 59

L_1 = moment on the side cc'.

L_2 = moment on the side ee'.

Let f_1 and f_2 be the forces on the sides.

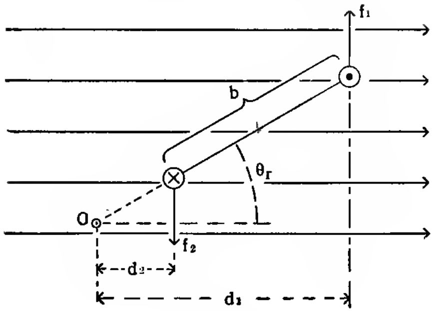


FIG. 59.

Let d_1 and d_2 be the moment arms of f_1 and f_2 respectively.

$$L = L_1 + L_2$$

$$L_1 = f_1 d_1$$

$$L_2 = f_2 d_2$$

$$f_1 = l\beta i$$

$$f_2 = -\, l\beta i$$

$$L = l\beta i(d_1 - d_2)$$

$$d_1 - d_2 = b\cdot\cos\theta_r$$

$$L = l\cdot\beta\cdot i\cdot b\cdot\cos\theta_r$$

$$l\cdot b = A \qquad \text{Area enclosed by coil.}$$

(405) $$L = A\cdot\beta\cdot i\cdot\cos\theta_r \quad \text{For one turn.}$$

(406) $$L = n\cdot A\cdot\beta\cdot i\cdot\cos\theta_r \quad \text{For } n \text{ turns.}$$

This is the expression for the moment of the force, due to the current, tending to rotate a rectangular coil about an axis which is parallel to one side of the coil and perpendicular to the field in which the coil is placed. It is equal to the continued product of the number of turns, by the area of the coil, by the magnetic induction through that area, by the current in the coil, by the cosine of the angle between the direction of the field and plane of the coil.

D'Arsonval System.—In this system a rectangular coil, of the preceding type, is suspended by an elastic fiber. By Hooke's law

$$L' = -K'\varphi \qquad (407)$$

See Fig. 58.

In which φ = the angle through which the fiber is twisted from the initial position of the coil, *gh*.

K' = moment of torsion of the fiber.
L' = moment of the restoring force.

For equilibrium the sum of the moments must equal zero. ∴ The moment of the restoring force plus the moment of the force due to the current (406), must equal zero.

$$L + L' = 0 \qquad (408)$$

$$n \cdot A \cdot \beta \cdot i \cdot \cos\theta_r = K'\varphi \qquad (409)$$

From (406) and (407).

$$\therefore \quad i = \frac{K'}{n \cdot A \cdot \beta \cdot \cos\theta_r} \cdot \varphi \qquad (410)$$

If

$$\cos\theta_r \doteq 1$$

(411) $$i \doteq \frac{K'}{n\cdot A\cdot\beta}\cdot\varphi$$

If, in addition to the above, the angular deflection is small, the angle may be taken as proportional to the deflection.

Let D = deflection as observed. See appendix on method of telescope and scale.

K'' = constant of proportionality.

(412) $$\varphi = K''\cdot D$$

If all the constant terms are combined in one term, K,

(413) $$i = K\cdot D$$

∴ For small deflections, the current is proportional to the deflection. The constant of proportionality may be determined by the principle of Ohm's law, thus calibrating the instrument by a known current. The latter is calculated in terms of standard resistances and e.m.f. of a standard cell.

Weston Type.—The Weston instrument is a portable type of the D'Arsonval galvanometer. In this type the coil is suspended by jewel bearings, and in a radial field. The elastic force of restitution is due to two spiral springs which also serve the purpose of leads to the moving coil. The radial field is obtained by the introduction of soft iron pole-pieces and a soft iron core. Fig. 60. The coil is arranged to rotate

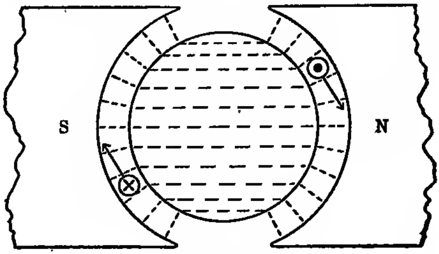


FIG. 60.

about the latter. In this device θ_r is constantly kept equal to zero as φ varies through a wide range, *i. e.*, of about 90 degrees.

$$\therefore \quad \cos \theta_r = 1$$

The angle φ can be measured accurately by means of a pointer and graduated scale. Let i_c = current in the moving coil.

$$i_c = K \cdot D \tag{414}$$

In which D = deflection. If the instrument is properly adjusted, it is the reading on the graduated scale.

Ammeters and Volt-Meters.—In order that an instrument of the Weston type may be used to measure a wide range of currents, a system of shunts is made use of. The ammeter with its shunt is connected in series with the circuit.

$$i = i_s + i_c$$

$$i_c R_c = i_s R_s \tag{311}$$

$$i_s = \frac{R_c}{R_s} i_c$$

$$i = \frac{R_c + R_s}{R_s} i_c$$

$$i_c = K \cdot D \tag{413}$$

(414) $$i = K \frac{R_c + R_s}{R_s} D$$

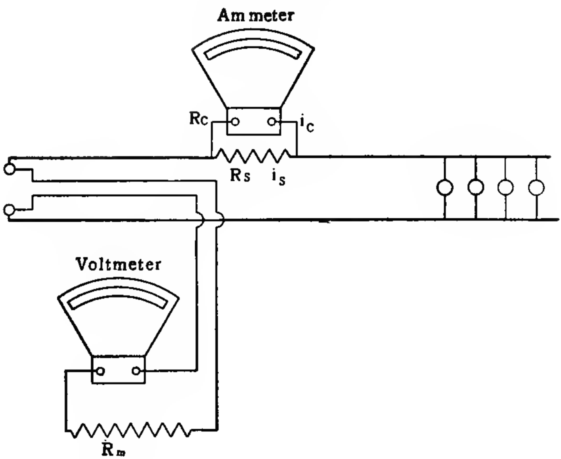


FIG. 61.

In which i = current in the main circuit or line.

K = a constant depending upon the moment of torsion of the fiber, number of turns in the coil, etc.

R_c and i_c are the resistance and current in the coil.
R_s and i_s are the resistance and current in the shunt.
D = angular deflection.

By use of different shunts, the relation between the current and scale reading can be varied through a wide range. If it is desired to multiply the scale-reading by a whole number, n, to obtain the value of the current in the line, the value of the shunt is calculated as follows:

$$i = ni_c \tag{415}$$

$$n = \frac{R_c + R_s}{R_s} \tag{416}$$

$$R_s = \frac{R_c}{n - 1} \tag{417}$$

In the voltmeter a series resistance, R_m, Fig. 61, is used. The voltmeter with its series resistance is placed in parallel with the circuit.
Let R_c and i_c be the resistance and current in the coil of the instrument.

$$e = (R_m + R_c)i_c$$

$$i_c = K \cdot D \tag{413}$$

$$e = K(R_m + R_c) \cdot D \tag{418}$$

By use of different resistances in series, the relation between the voltage and scale reading can be varied through a wide range. For this reason R_m is called a "multiplier." If it is desired to multiply the scale reading by a whole number, n, the value of the multiplier is calculated as follows:

$$e = ne_c \tag{419}$$

In which e_c = difference of potential between the ends of the coil

$$e_c = R_c i_c$$

$$R_m = (n - 1)R_c \tag{420}$$

The same instrument may be used as an ammeter and as a voltmeter through a wide range of values, by use of a system of shunts and multipliers.

PROBLEMS.

100. An ammeter requires 0.006 ampere through the coil, for a scale deflection of six units. The resistance of the coil is measured and found to be 4 ohms. What must be the resistance of the shunt so that one ampere in the main circuit shall give unit deflection on the scale? Ans. 0.00400+ ohm.

101. An ammeter is connected in series with an absolute tangent galvanometer. The galvanometer consists of a single coil 30 cm. in diameter. It is set up in a field of "horizontal intensity" 0.18 dyne per unit pole. The deflection of the galvanometer is $\tan^{-1} 0.0105$ when the deflection of the ammeter is 4.5+ divisions. If the resistance of the coil is 5 ohms, what must be the resistance of the shunt so the ammeter will be deflected one division for one ampere in the main circuit?

Ans. 0.05 ohm.

102. The resistance coil of a Weston milli-voltmeter is 10 ohms and the resistance of the moving coil part is 8 ohms. It is found that 0.2 volt gives a reading of 200 divisions. What current through the coil deflects it one division? If the 10 ohms be removed, what resistance must replace it so 100 volts will give a deflection of 110 divisions?

Ans. 0.0000556 ampere.
18,000 ohms.

103. A Weston volt-meter has a coil resistance of 1.8 ohms, and a series resistance of 998.2 ohms. One volt is indicated by a deflection of one division. How can the same coil be shunted for use as an ammeter so one ampere in the main circuit causes a deflection of 100 divisions? Ans. 0.2 ohm.

104. There are 60 numbered divisions on the scale of a certain Weston instrument. The resistance of its coil is 5 ohms. A current of 3/50 ampere through the coil gives a full scale deflection. What auxiliary resistance is necessary to use the instrument as:

(*a*) A volt-meter reading volts direct?
(*b*) An ammeter reading amperes direct?

Ans. Shunt of 0.00500+ ohm for ammeter.
Multiplier of 995 ohms for voltmeter.

CHAPTER XII.

ELECTRO-MAGNETIC INDUCTION.

If part of a circuit having a sensitive galvanometer in series, Fig. 62, is moved across a magnetic field,

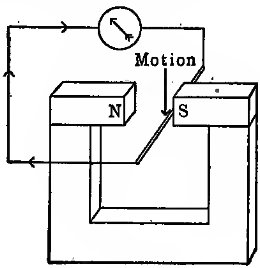


FIG. 62.

there will be a deflection in the galvanometer. From this fact and Ohm's law it is inferred that an e.m.f. is induced in the conductor. This process is called electro-magnetic induction. The experiment is due to Faraday, who showed experimentally that the induced e.m.f. is proportional to the rate at which the conductor cuts across the field. The direction of the induced e.m.f. is such that the force due to the induced current opposes the motion of the conductor. This statement is known as Lenz's law. The electrical work done by the induced e.m.f. and induced current, is numerically equal to the work done, in moving the conductor against the force due to the induced current. If the conductor is moved during an element of time, dt, the electrical work done is:

(425) $$dw = e \cdot i \cdot dt$$ From (300) and (309).

If the distance moved in the time dt, is dx, the mechanical work done is:

(426) $$dw = f_X \cdot dx$$

In which

$$f_X = \int i \cdot \beta \cdot \sin \theta \cdot \cos \varphi_X \cdot dl \qquad (401)$$

(427) $$dw = \int i \cdot \beta \cdot \sin \theta \cdot \cos \varphi_X \cdot dl \cdot dx$$

From the law of conservation of energy, dw must be equal in (425) and (427).

(428) $$e \cdot dt = \int \beta \cdot \sin \theta \cdot \cos \varphi_X \cdot dl \cdot dx$$

The right-hand member may be expressed in terms of the flux through the element of surface swept out by the conductor, as follows:

$$\varphi = \beta \cdot S \cdot \cos \alpha \qquad (38)$$

$$S' = S \cdot \cos \alpha \qquad \text{Fig. 63.}$$

$$\therefore \quad \varphi = \beta \cdot S'$$

$$S' = \overline{ab} \times \overline{cd}$$

$$\overline{ab} = dl \sin \theta$$

$$\overline{cd} = dx \cos \varphi_X$$

$$S' = dl \cdot \sin \theta \cdot dx \cdot \cos \varphi_X$$

If $d\varphi$ = the flux through the element of surface swept out by the conductor, instead of φ,

(440) $$d\varphi = \int \beta \cdot \sin\theta \cdot \cos\varphi_X \cdot dl \cdot dx$$

(445) $$\therefore \quad e \cdot dt = d\varphi$$

(450) $$\therefore \quad e = \frac{d\varphi}{dt}$$

The e.m.f. induced in a conductor at any instant is numerically equal to the ratio of the flux though the

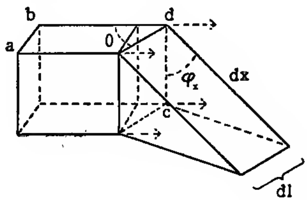


FIG. 63.

element of surface swept out by the conductor, to the element of time. This is called the *instantaneous value* of the induced e.m.f. The average value of the induced e.m.f. may be defined as the ratio of the total normal induction or flux, through the surface swept out by the conductor, to the time.

(451) $$E_{av} = \frac{\varphi}{t}$$

In which E is in e.m.u. when φ is in e.m.u. and t in seconds. When E is in volts, *i. e.*, in practical units, the flux, must be in practical units. ∴ The practical unit of flux = $(10)^8$ e.m.u. of flux.

Applications.—In the case of a straight conductor in a uniform field, the terms shown in parentheses are the same for all elements.

$$d\varphi = \int (\beta \cdot \sin\theta \cdot \cos\varphi_X \cdot dx) \cdot dl \qquad (440)$$

$$\therefore \quad d\varphi = \beta \cdot \sin\theta \cdot \cos\varphi_X \cdot dx \cdot \int dl$$

for a straight conductor.

(452) $$d\varphi = l \cdot \beta \cdot \sin\theta \cdot \cos\varphi_X \cdot dx$$

(453) $$\frac{d\varphi}{dt} = l \cdot \beta \cdot \sin\theta \cdot \cos\varphi_X \cdot \frac{dx}{dt}$$

(454) $$e = l \cdot \beta \cdot \sin\theta \cdot \cos\varphi_X \cdot v$$

In which v = velocity with which the conductor is moved. If the direction of field, conductor and motion are mutually perpendicular,

(455) $$e = l \cdot \beta \cdot v$$

Closed Circuits.—Let *abc*, Fig. 64, represent a closed conductor forming a closed curve in a magnetic field.

Let φ_1 = total normal induction through a surface bounded by the conductor.

Let the conductor be moved an elementary distance or the form of the curve changed by an infinitesimal amount.

Let φ_2 = total normal induction through the surface bounded by the conductor after the change.

(456) $$d\varphi = \varphi_2 - \varphi_1$$

In which $d\varphi$ = the total normal induction through the surface swept out by the conductor during the change. It is represented by the shaded portion in the figure.

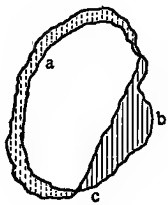


FIG. 64.

$\varphi_2 - \varphi_1$ = change of total normal induction through the surface enclosed by the conductor. If this change takes place in the time *dt*,

(457) $$e = \frac{\varphi_2 - \varphi_1}{dt}$$ From (450) and (456).

This may be written in the same form as (450).

(458) $$e = \frac{d\varphi}{dt}$$

In which φ = total normal induction through the surface bounded by the conductor, or the total flux enclosed by the conductor.

The e.m.f. induced in a conductor forming a closed curve, is numerically equal to the time rate of change of the flux enclosed by the conductor.

Coefficients of Induction.—If, instead of moving the conductor across the magnetic field, the conductor is fixed and the magnetic field moved, Fig. 65, an e.m.f.

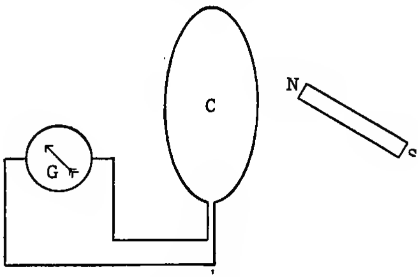


FIG. 65.

is induced. The direction of the induced e.m.f. is such that the force due to the induced current, opposes the relative motion of the field and conductor. The essential characteristic of electro-magnetic induction is the change in the relation of the conductor to the distribution of the intensity of the magnetic field. This change, in the relation of the conductor to the distribution of the magnetic field, may be due to the variation of a current in a second conductor, Fig. 66. Even though the geometrical relations of two conductors or a number of conductors remain fixed, if the current in one conductor changes, the distribution of the flux, due to that current, changes its relation

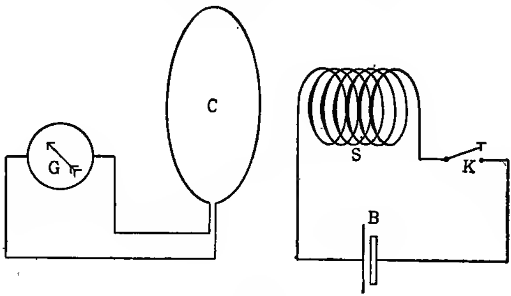


FIG. 66.

to the conductor itself and to every other conductor in space. The value of the induced e.m.f. in each conductor, is the rate at which this change, in the relation of the flux to the conductor, takes place.

The ratio of the change of flux across a conductor, to the change of the current, to which the change of flux is due, is called a *coefficient of induction.* Coefficients of induction are of two kinds: the coefficient of *self-induction* and the coefficient of *mutual induction.*

Self-induction.—If the change of flux across the conductor is due to the change of current in the conductor itself, the coefficient is called self-induction or self-inductance. It is represented by the symbol L.

$$L = \frac{d\varphi_1}{di_1} \tag{460}$$

In which $d\varphi_1$ = change of flux across the conductor.

di_1 = change in current in the conductor.

If this change of flux takes place in the time dt we have:

(461) $$\frac{d\varphi_1}{dt} = L\frac{di_1}{dt}$$

(462) $$\therefore \quad e = L\frac{di_1}{dt}$$

In which e is called the e.m.f. of self-induction. Its direction in the conductor is such that it always opposes the change of current.

If the geometrical relations remain constant and μ is constant, there is a constant relation between the current and flux due to that current. L is therefore constant and

(463) $$\therefore \quad L = \frac{\varphi_1}{i_1}$$

Mutual Induction.—If the change of flux across the conductor is due to the change of current in a second conductor, the coefficient is called mutual induction or mutual inductance. It is represented by the symbol M. If the coefficient of mutual inductance is large, the circuits are said to be *magnetically related.*

(465) $$M = \frac{d\varphi_1}{di_2}$$

In which $d\varphi_1$ = change of flux across the first conductor.

di_2 = change of current in the second conductor.

Unit Inductance.—The *unit of inductance* is called the e.m.u. of inductance, if the flux and the current are expressed in e.m.u. The unit of inductance is called the "practical unit of inductance" or the "*henry,*" if the unit of flux is the practical unit of flux or $(10)^8$ e.m.u. of flux, and the unit of current is the ampere.

$\therefore$ One henry = $(10)^9$ e.m.u. of inductance.

(466) (Number of henries)$(10)^9$ = (Number of e.m.u. of inductance).

One henry is an inductance of such a magnitude that a variation of one ampere produces a variation of $(10)^9$ e.m.u. of flux through the area bounded by the conductor in which the e.m.f. is induced. This is a large unit of inductance. The one thousandth part of the henry, called the "milli-henry," is in common use.

(467) (Number of milli-henries)$(10)^6$ = (Number of e.m.u. of inductance).

If the change of flux, in equation (465), takes place in the time dt,

(468) $$\frac{d\varphi_1}{dt} = M\frac{di_2}{dt}$$

(469) $$\therefore \quad e = M \frac{di_2}{dt}$$

From (468) and (457).

In which e is called the e.m.f. of mutual inductance.

If the geometrical relations of the two circuits remain constant and μ is constant, there is a constant relation between the flux cutting across one conductor and the current, in the second conductor, to which this flux is due.

(470) $$M = \frac{\varphi_1}{i_2}$$

The coefficients of induction for conductors in air, therefore depend upon the geometrical forms of the conductors only and can be calculated independent of any other electrical quantity.

PROBLEMS.

110. A horizontal wire one meter long, at right angles to the earth's magnetic meridian, was moved vertically with a velocity of 1,000 cm. per second. The horizontal intensity of the earth's field being 0.2 dyne per unit pole, what e.m.f. was induced, in volts? What flux was cut in two seconds? In one second?

CHAPTER XIII.

MAGNETIC CIRCUITS.

Let *abc*, Fig. 67, represent any closed circuit. Let P be any point. Let m be a magnetic pole at P. As m is moved an infinitesimal distance, the flux through a surface, enclosed by *abc*, varies. An e.m.f. is induced and the work done is:

$$dw = e \cdot i \cdot dt \tag{425}$$

In which e = induced e.m.f.

i = current in the conductor.

(471) $$w = \int eidt$$

$$e = \frac{d\varphi}{dt}$$

(472) $$w = \int i \cdot d\varphi$$

$$d\varphi = \beta \cos \alpha \, dS$$

In which α is the angle between β and the normal to the surface at that point.

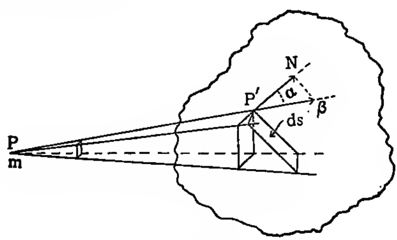


FIG. 67.

$$\beta = \frac{m}{r^2} \tag{51}$$

(473) $$w = \int m \cdot i \cdot \frac{\cos \alpha \cdot dS}{r^2}$$

Let $d\omega$ = elementary solid angle subtended by dS.

(474) $$d\omega = \frac{dS \cdot \cos \alpha}{r^2}$$

(475) $$w = \int_{\omega_1}^{\omega_2} m \cdot i \cdot d\omega$$

From (473) and (474).

If both m and i are constant,

(476) $$w = mi \int_{\omega_1}^{\omega_2} d\omega$$

If m is moved by any path back to its initial position,

(477) $$\omega_2 = \omega_1 + 4\pi \cdot N$$

In which N = the number of times the path of m interlinks the conductor.

(478) $$w = 4\pi \cdot N \cdot m \cdot i$$

From (476) and (477).

This expression is known as Weber's law.

If in this equation

$$N = 0$$

(479) $$w = 0$$

The pole, m, may be moved in a closed curve without interlinking the conductor. In this case the algebraic sum of the work done is equal to zero.

Anchor Ring.—If a long solenoid is bent back upon itself in the form of a ring, it is called an anchor-ring, Fig. 68.

Let N = number of turns on the anchor-ring.

i = current.

$\mathcal{H}$ = intensity of field, due to i, in the circular axis of the anchor-ring.

If a pole, m, is moved along the circular axis one complete revolution, it goes about each turn of the conductor once.

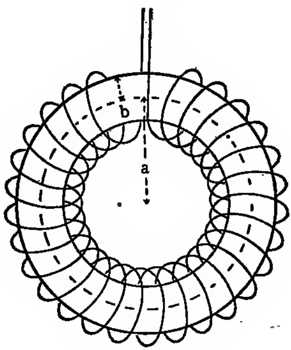


FIG. 68.

$$w = 4\pi Nmi \tag{478}$$

But

$$w = f \cdot l$$

$$f = m\mathcal{H}$$

(480) $$w = m \cdot \mathcal{H} \cdot l$$

In which $l =$ length of the circular axis of the anchor-ring.

$\mathcal{H} =$ field due to the current.

$m =$ pole moved along the axis.

(481) $$\mathcal{H} = \frac{4\pi Ni}{l}$$

From (478) and (480).

Average Intensity in the Core.—Starting at any point in the core of the anchor-ring, and completing a cycle by any path through the core, parallel to the axis.

$$w = 4\pi Nmi \tag{478}$$

Let $\mathcal{H}' =$ intensity along the path.

$l' =$ length of the path.

(482) $$w = m\mathcal{H}'l'$$

(483) $$\mathcal{H}'l' = \mathcal{H}l$$

From (478), (480) and (482).

$\mathcal{H}l$ along any path through the core is the same. Since l along the axis is the average length of path, $\mathcal{H}$ along the axis is the average intensity inside the core of the anchor-ring.

Rowland's Law.—If the permeability of the core of the anchor-ring is equal to μ,

$$\beta = \mu\mathcal{H} \tag{50}$$

(485) $$\beta = \frac{4nN\mu i}{l}$$

From (481) and (50).

If φ = flux through the cross section of the core,

$$\varphi = \beta \cdot S$$

(486)
$$\varphi = \frac{4\pi Ni}{\left(\frac{l}{\mu S}\right)}$$

This expression is called Rowland's law. It is said to be analogous to Ohm's law, but the analogy is purely mathematical.

Magneto-motive Force.—Magneto-motive force is defined as the work per unit pole in carrying a pole about a current. It is represented by the symbol $\mathfrak{M}$.

(490)
$$\mathfrak{M} = \frac{w}{m}$$

It is the same for every closed cycle along the magnetic whorl, or magnetic circuit.

(491)
$$\mathfrak{M} = 4 \cdot \pi \cdot N \cdot i$$

From (478) and (490).

The expression $l/\mu S$ is called *reluctance*. It is represented by the symbol $\mathfrak{R}$. $\therefore$ Equation (486) can be written:

(492)
$$\varphi = \frac{\mathfrak{M}}{\mathfrak{R}}$$

i. e., the flux, magneto-motive force, and the reluctance have the same mathematical relation to each other in a magnetic circuit, as the current, electro-motive force, and resistance have in an electric current. In this analogy, l/μ bears the same relation to the reluctance as the specific resistance, γ, bears to the resistance. The latter is a constant for constant temperatures and is independent of the e.m.f., μ however, is not a constant. It is a function of the magneto-motive force (m.m.f.).

If the magnetic circuit is composed of parts of different permeability, the flux through every complete cross-section of the magnetic circuit is the same, *i. e.*, the resultant induction due to the magnetizing force of the current and that due to the magnetism induced on the surface of separation between the parts of different permeability, is continuous through the surface. This may be shown as follows. If the magnetic circuit is cut across a closed electric circuit in which the current is maintained constant, the flux enclosed by the circuit increases from zero to a maximum and back again to zero in the opposite direction. Therefore the work done is, by Weber's law, equal to zero.

$$w = \int i \cdot d\varphi \tag{472}$$

$$w = i\int d\varphi$$

since the current is constant. But

$$w = 0$$

$$\therefore \quad \int d\varphi = 0$$

If the magnetic circuit is composed of substances of different permeabilities in series,

(493) $$\mathfrak{R} = \Sigma\mathfrak{R}$$

For a small air gap in an iron circuit:

(494) $$\mathfrak{R} = \frac{l_1}{\mu_1 S} + \frac{l_2}{S}$$

In which l_1 = length of the iron circuit.
l_2 = length of the air gap.

Rowland's law for such a circuit may be written:

(495) $$\varphi = \frac{4\pi Ni}{\dfrac{l_1}{\mu_1 S} + \dfrac{l_2}{S}}$$

Electrostatic Flux and Magnetic Flux.—The relations of the quantities involved in Rowland's law (486), are more nearly analogous to those involved in electrostatic flux.

$$q = C \cdot e \qquad (200)$$

$N = 4\pi Ce.$ From (137) and (200).

For plate condenser,

$$C = \frac{\kappa A}{4\pi d} \qquad (205)$$

(496) $$N = \frac{e}{\left(\dfrac{d}{\kappa A}\right)}$$

Compare the physical relations of these quantities with those existing between the quantities in Rowland's law. See (1) and (100).

Inducing Field.—If a substance of permeability μ_1 is placed in a magnetizing field, $\mathcal{H}$, the intensity of magnetization will be such that the resultant induction, through the surface of separation, is the same on both sides of the surface.

$$\mu_1\mathcal{H} + \beta_1 = \mu_2\mathcal{H} + \beta_2$$

In which β_1 and β_2 are the inductions due to the induced magnetism. For components normal to the surface of separation:

$$(497) \qquad (\mu_1\mathcal{H} + \beta_1)\cos\alpha = (\mu_2\mathcal{H} + \beta_2)\cos\alpha$$

But

$$\beta_2 \cos\alpha = \beta_1 \cos\alpha + 4\cdot\pi\cdot\mathfrak{I} \qquad (71)$$

$$(498) \qquad \therefore \quad \mu_1\mathcal{H}\cos\alpha = \mu_2\mathcal{H}\cos\alpha + 4\cdot\pi\cdot\mathfrak{I}$$

If the surface is perpendicular to the magnetizing force:

$$\cos\alpha = 1$$

$$(499) \qquad \therefore \quad \mu_1\mathcal{H} = \mu\mathcal{H}_2 + 4\cdot\pi\cdot\mathfrak{I}$$

If $\mu_2 = 1$ in air, and $\beta = \mu_1\mathcal{H}$,

$$(500) \qquad \beta = \mathcal{H} + 4\cdot\pi\cdot\mathfrak{I}$$

This represents the numerical relation generally used in work on magnetic induction.

Magnetization Curves.—If the intensity, $\mathcal{H}$, of the magnetizing field, in which a piece of soft iron is placed, is increased from zero the iron is magnetized. The intensity of magnetization increases with the intensity of the magnetic field, $\mathcal{H}$, but is not directly proportional to it.

As $\mathcal{H}$ is increased a value is reached at which the magnetization becomes constant and remains so after further increase of $\mathcal{H}$. This particular value of $\mathcal{H}$ is called the "saturation value."

Let μ = permeability of the iron.
β = induction.

$$\beta = \mu\mathcal{H} \qquad \text{By definition (33).}$$

The curve, I, Fig. 69, representing the variation of the induction with the intensity, as $\mathcal{H}$ increases from zero to the saturation value, is called the "magnetization curve" or "saturation curve." If, now, the intensity of the magnetizing field is gradually decreased to zero, the induction is decreased, but not to zero, II, Fig. 69. The corresponding values of β are larger on the descending curve. The iron is not completely demagnetized. The remaining magnetism is called "residual magnetism." If the intensity of the magnetizing field is gradually increased from zero to the same saturation value in the opposite direction, the induction gradually decreases, passing through zero to the maximum value in the opposite direction, III,

Fig. 69. If now the magnetizing field is gradually changed to zero, reversed, and increased to the first maximum, the curves, IV and V, Fig. 69, are traced. Curves II and III are symmetrical with IV and V.

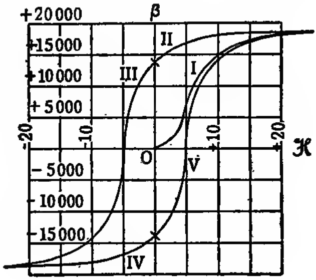


FIG. 69.

As the intensity of field executes a cycle of values, the intensity of magnetization executes a cycle but "lags" behind the intensity of the magnetizing field. This lag is called *hysteresis* and the curve II, III, IV, V, is called a *hysteresis loop.*

Experimental Determination.—These curves may be experimentally determined by the relation

$$\beta = \mathcal{H} + 4\pi\mathfrak{J} \tag{500}$$

by plotting the corresponding values of β and $\mathcal{H}$. The intensity $\mathcal{H}$, is generally obtained by means of a solenoid or anchor-ring. Its value can be expressed in terms of the current and the constants of the coil.

$$\mathcal{H} = \frac{4\pi Ni}{l} \qquad (481) \text{ or } (248)$$

$\mathfrak{I}$ may be determined in terms of the magnetic field due to the induced poles, by means of a magnetometer. β can then be calculated in terms of $\mathcal{H}$ and $\mathfrak{I}$ and the corresponding values plotted.

PROBLEMS.

120. An anchor-ring having a circular axis of 20 cm. radius is wound with 600 turns of wire. If the current in the wire is 2 amperes what is the intensity of the inducing field?

121. Calculate the induction in an anchor-ring having a circular axis 18 cm. in diameter and wrapped with 800 turns of wire: the current in the wire being 0.2 ampere and the permeability of the core 900 e.m.u.?

122. What current is necessary to produce an induction of 8,000 e.m.u. in an iron core of permeability 1,000 and wrapped with 50 turns of wire per cm. of length of axis?

123. Calculate the total magnetic flux through the core of an anchor-ring, from the following data: permeability = 1,800 e.m.u., cross-section = 50 sq. cm., turns per cm. length of axis = 20, current = 0.5 ampere.

124. A permanent magnet, 30 cm. long, is magnetized to an intensity of 700 e.m.u. The "faces" are perpendicular to the axis of the magnet and have the dimensions 1 cm. $\times$ 1/2 cm. Calculate the pole strength.

125. An iron magnet of cross-section 3,000 sq. cm., and 100.8 cm. length of axis, is made in two parts separated by air gaps each 4 mm. long. The permeability of the iron is 1,800 e.m.u. Calculate the total reluctance.

126. What current must be sent through the coil of an anchor-ring 8 cm. in radius to give 6,000 e.m.u. of induction, if there are 450 turns in the coil and the permeability of the core is 900?

127. Calculate the total flux through a cylinder of iron of permeability 1,200, if 30 turns per unit length be wrapped about it. The current through the coil is 0.6 ampere and the core is 5 cm. in radius.

CHAPTER XIV.

ROTATING COIL IN A UNIFORM FIELD.

Let *abc*, Fig. 70, represent a plane coil free to rotate about an axis, O, perpendicular to the uniform field and so placed that the plane of the coil is parallel to the axis of rotation.

Let A = area enclosed by the coil.

θ = angle between the direction of the field and plane of the coil.

φ = magnetic flux through the coil at any instant.

(550) $$\varphi = A \cdot \beta \cdot \sin \theta$$

If the coil is rotating with a uniform angular velocity, ω,

$$\theta = \omega t$$

(551) $$\varphi = A \cdot \beta \cdot \sin \omega t$$

(552) $$\frac{d\varphi}{dt} = \omega A \cdot \beta \cdot \cos \omega t \qquad \text{From (551).}$$

(553) $$e = \omega A \cdot \beta \cdot \cos \omega t \quad \text{From (450) and (552).}$$

If there are n turns in series in the coil,

(554) $$e = n \cdot \omega \cdot A \cdot \beta \cdot \cos \omega t$$

When the plane of the coil is parallel to the field the rate of cutting the flux is greatest *i. e.*, when

$$\cos \omega t = 1$$

$$e = E_{\max}$$

(555) $$E_{\max} = n\omega A\beta$$

(556) $$e = E_{\max} \cos \omega t$$

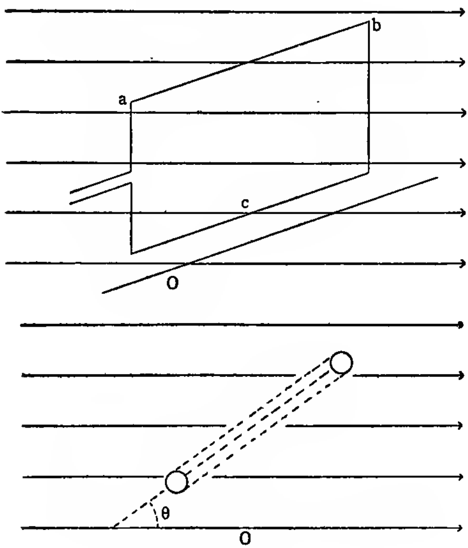


FIG. 70.

In the last two equations E is expressed in electromagnetic units. If E is to be expressed in volts,

(557) $$E_{\max} = \frac{n\omega A\beta}{(10)^8}$$

Equation (554) represents the instantaneous value of the induced e.m.f. This relation is called a harmonic function and may be shown graphically by plotting the values of e with corresponding values of ωt. See Fig. 71.

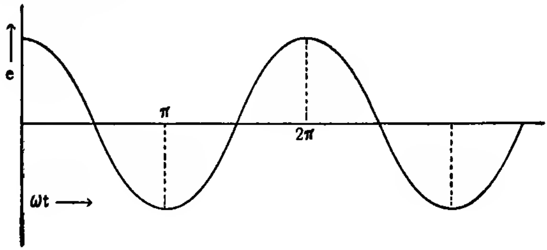


Fig. 71.

Since the values of e are alternately positive and negative, but can be arranged in pairs of the same numerical values, the average value of the induced e.m.f. is equal to zero.

Rectification.—Let the terminals of the plane coil be connected to two half rings which are mounted upon the shaft of the axis of rotation but insulated from

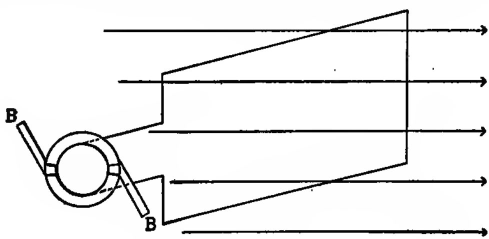


Fig. 72.

each other. By means of two brushes these half-rings are connected to an external circuit. The brushes are so adjusted that they pass from one of the half-rings to the other as the value of the induced e.m.f. passes through zero. This causes the e.m.f. at the brushes to be always in the same direction. It may be plotted with the corresponding values of ωt, as shown in Fig. 73.

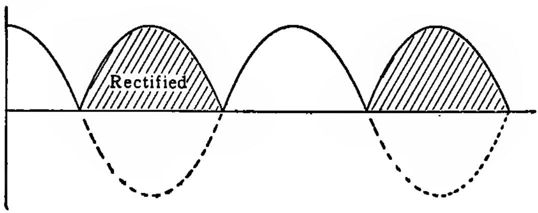


FIG. 73.

Average E.—The average value of this curve is not equal to zero. Its value is the average value for the first quarter turn as plotted in Fig. 73. As the plane of the coil is rotated from a position perpendicular to the field to the position parallel to the field, the flux enclosed by the conductor varies from $A\beta$ to 0. If the coil is rotating f revolutions per second, the time necessary for $\frac{1}{4}$ revolution is $1/4f$.

Let φ = the maximum value of the flux enclosed by the coil.

$$\varphi = A\beta$$

$$t = \frac{1}{4f}$$

(558) $$E_{av} = 4A\beta f$$ From (451).

If there are n turns in series in the coil,

(560) $$E_{av} = 4nA\beta f$$

The relation between the average and the maximum e.m.f. can be derived from equations (555) and (560)

$$E_{max} = n\omega A\beta \qquad (555)$$

$$\omega = 2\cdot\pi\cdot f$$

(561) $$\therefore \quad E_{av} = \frac{2}{\pi}E_{max}$$

Measurement of a Varying Electro-motive Force. —If the poles of a varying e.m.f. are connected to the plates of a weight-electrometer, there will be at any instant:

$$e = \sqrt{\frac{8\pi d^2}{A}\cdot f} \qquad (154)$$

$$\therefore \quad e^2 = \frac{8\pi d^2}{A}\cdot f$$

Let

$$\frac{8\pi d^2}{A} = K, \text{ a constant}$$

$$e^2 = Kf$$

for the successive values of e.

(562) $$\frac{e_1^2 + e_2^2 + \cdots e_n^2}{n} = K\frac{f_1 + f_2 + \cdots f_n}{n}$$

The mean square value of the difference of potential is proportional to the average value of the force. The first member of the equation is always greater than zero, since every term is always positive, unless each term is equal to zero. ∴ The average force is > 0. Let

$$\sqrt{\frac{e_1^2 + e_2^2 + \cdots e_n^2}{n}} = E_{\text{eff}} \tag{563}$$

In which E_{eff} is called the "effective e.m.f." In general the effective value of a varying quantity is defined as the square root of the mean square of the quantity. Let

$$\frac{f_1 + f_2 + \cdots f_n}{n} = F_{\text{av}} \tag{564}$$

$$E_{\text{eff}} = \sqrt{\frac{8\pi d^2}{A} \cdot F_{\text{av}}} \tag{565}$$

∴ The weight electrometer may be used to measure the effective value of a varying e.m.f.

Relation of Effective and Maximum Values.—In a pure sine function the ratio of the effective to the maximum e.m.f. may be determined by the method shown in Fig. 74.

Divide every half wave-length into n equal parts, numbered consecutively, 1, 2, 3, etc., $\cdots, \frac{n}{2}, \frac{n}{2} + 1, \frac{n}{2} + 2$, etc. The number of the middle division is $n/2$.

For any division, K, in the first quarter wave-length, there will be a corresponding division $n/2 + K$, in the second quarter wave-length. The difference of

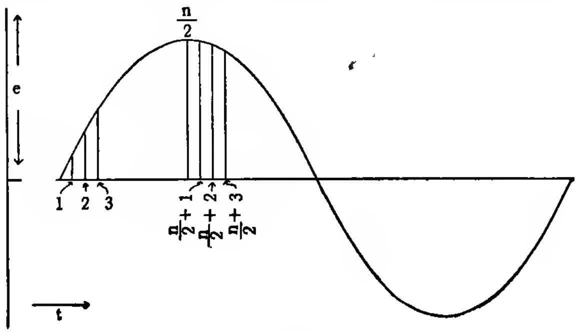


FIG. 74.

phase (simple harmonic motion) is one quadrant or $\pi/2$ radians for each pair of divisions. In the first quarter wave-length the expression for e will be:

(566) $$e_\kappa = E_{\max} \sin \omega t \qquad (556)$$

The value of e for the corresponding division on the next quarter wave-length will be:

(567) $$e_{n/2+\kappa} = E_{\max} \cos \omega t$$

Square each and add the $\frac{n}{2}$ pairs of e^2:

(568) $$\frac{e_1^2 + e_2^2 + e_3^2 + \cdots e_n^2}{n} = \frac{\frac{n}{2} E_{\max}^2}{n}$$

(569) $$E_{\text{eff}} = E_{\max} \tfrac{1}{2} \sqrt{2}$$

From (563) and (568).

PROBLEMS.

135. At what frequency must a single coil, 18 cm. long and 11 cm. wide, be rotated about an axis perpendicular to a uniform field of 12,000 dynes per unit pole, to give an everage e.m.f. of 2.2 volts? What is the maximum e.m.f. induced?

136. A coil of 300 sq. cm. area is rotated at 1,200 r.p.m. about an axis perpendicular to a uniform field of 9,000 e.m.u. of induction? Find the maximum e.m.f. induced and the value of the e.m.f. 1/120 sec. after the coil passes a position parallel to the direction of the field; 1/80 sec. after this position.

137. A coil, 25 cm. × 12 cm., is rotated with a constant speed of 1,260 r.p.m., about an axis perpendicular to a uniform field. An average e.m.f. of 3.1 volts is generated in a single turn. What is the magnetic induction in this field?

138. The resultant direction of the earth's field is 60°, *i. e.*, the dip is 60°, and its intensity is 0.4 dyne per unit pole. A coil of 25 turns, 40 × 20 cm., is rotated uniformly 30 times per second upon a vertical axis. Find the average e.m.f. induced in the coil.

139. A coil of wire consists of 50 turns of wire in the form of a circle, 40 cm. in diameter. It is rotated 20 times per second about a vertical axis. Find the maximum e.m.f., in volts, induced in the coil if the horizontal component of the earth's field is 0.18 dyne per unit pole.

140. A coil, 18″ long and 12″ wide, is rotated uniformly at 1,800 r.p m. in a field of 40,000 e.m.u. of induction. Find the maximum, average, and effective e.m.f. and the instantaneous value 0.01 sec. after it passes through the point of zero e.m.f.

141. How fast must a single coil of 400 sq. cm. area be rotated, perpendicularly to a uniform field of 8,000 e.m.u. of induction, to produce an effective e.m.f. of 2.828 volts?

142. A circular disk of copper, radius 40 cm., is rotated about an axis perpendicular to its plane. A uniform field, perpendicular to the disk, is of strength 20,000 e.m.u. of induction. Find the number of revolutions per second necessary to induce an e.m.f. of 4 volts between the center and the circumference of the disk.

CHAPTER XV.

COEFFICIENTS OF INDUCTION.

Direction of Induced E.M.F.—Let AB and CD, Fig. 75, be two parallel conductors. If the current in AB increases, the change in the relation of the flux to

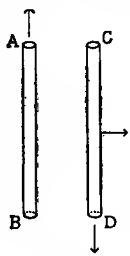


FIG. 75.

CD will be, as if the conductors were brought nearer together. (See Lenz's law.) The direction of the force on CD is outward and the direction of the induced e.m.f. will be from C to D. If the current in AB decreases the direction of the force on CD, is inward and the direction of the induced e.m.f. is from D to C.

If AB and CD are parts of the same conductor they may be joined in two ways: (First) Fig. 76. The induced e.m.f. in CD, due to the increase of current in AB, is in the direction of the current in CD. (Second)

Fig. 77. The induced e.m.f. in CD, due to the increase in current in AB, opposes the current in CD. The self-induction of the first type is small and it is called a *non-inductive resistance.* The self-induction of the

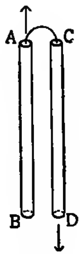


FIG. 76.

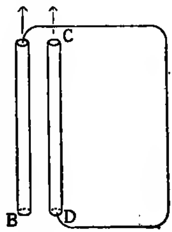


FIG. 77.

second type is large and it is called an *inductive resistance.* Since the induced e.m.f. is proportional to the rate of change of flux, its value is zero when the current is constant The maximum value of the current in a conductor, joined to a constant e.m.f., depends upon the resistance only but the time required to attain this maximum value depends upon the self-induction. Fig. 78 shows this relation of the current and time in inductive and in non-inductive resistances.

The difference between an inductive and a non-inductive resistance can be experimentally demonstrated by means of the apparatus shown by diagram in Fig. 79. The apparatus consists of two circuits joined to the same source of difference of potential, e.

One of these circuits contains a non-inductive resistance, like that shown in Fig. 76, in series with one coil of a differential galvanometer; the other circuit contains an inductive resistance. like that shown in Fig.

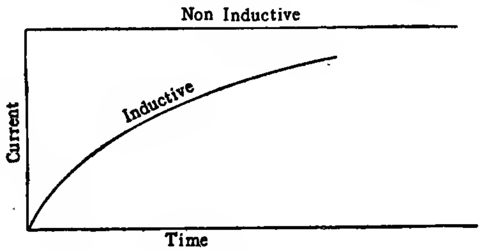


FIG. 78.

77, in series with the second coil of the differential galvanometer. The resistance of the two circuits is the same. The differential galvanometer consists of two similar coils so placed that the field at the needle, due to one coil, opposes that due to the other.

If both circuits are closed at the same instant by pressing the key, K, the current builds up more rapidly in the non-inductive circuit and the needle is momentarily deflected in the direction of the field due to this coil. Since the resistance of the circuits is the same, the maximum values of the two currents must be the same. After a short time the needle will come to rest. If now the circuits are broken at the same instant, the current in the non-inductive circuit dies down more rapidly and the needle is deflected in the direction of the field due to the inductive circuit. This experiment verifies the deductions of the preced-

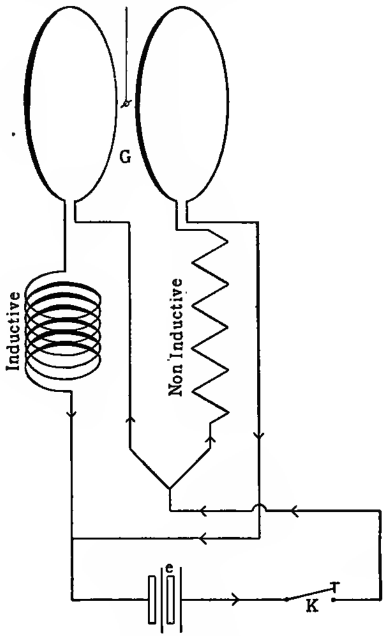


FIG. 79.

ing paragraph and demonstrates the difference in behavior of an inductive and of a non-inductive resistance.

Self-inductance of Solenoid.—A study of the ordinary solenoid (see Fig. 44) will reveal that it is wound inductively.

Let L = coefficient of self-induction of the solenoid.
$\dot{i}_1$ = current in the coil.
φ = flux cutting a single turn.
N_1 = number of turns.
φ_1 = flux through all turns.

$$L = \frac{\varphi_1}{\dot{i}_1} \qquad (463) \quad \text{If } u \text{ is constant.}$$

$$\varphi_1 = N_1 \varphi$$

$$\varphi = \beta \cdot S_1$$

$$\beta = \mu \mathcal{H} \qquad (50)$$

$$\mathcal{H} = \frac{4\pi N_1}{l_1} \cdot \dot{i}_1 \qquad (248)$$

$$(570) \qquad L = \frac{4\pi N_1^2 S_1 \mu}{l_1}$$

Where the value of μ is a function of the maximum value of the induction, β, and the character of the substance of which the core is made. The value to be used in equation 570, is determined from experimental data.

Coefficients of Mutual Inductance.—The coefficient of mutual inductance between a solenoid and a coaxial coil may be determined as follows:

Let gh, Fig. 80, represent part of a solenoid.
" cd represent a second, coaxial coil, placed at the middle of the long solenoid.

Let M = mutual inductance of the solenoid and the coil.

i_1 = current in gh.

φ = flux per single turn of cd. It is assumed to be constant for all turns of cd and ab.

φ_2 = flux through all turns of cd.

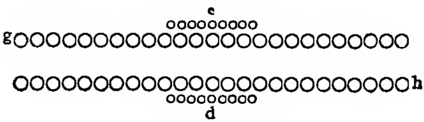


FIG. 80.

$$M = \frac{\varphi_2}{i_1}$$

(470) If μ is constant.

Let N_2 = number of turns in cd.

$$\varphi_2 = N_2\varphi$$

$$\varphi = \beta_1 S_1 \qquad (67)$$

In which S_1 is the area enclosed by a single turn of cd.

$$\beta = \mu\mathcal{H} \qquad (50)$$

$$\mathcal{H} = \frac{4\pi N_1}{l_1}\cdot i_1 \qquad (248)$$

Substituting:

(571) $$M = \frac{4\pi N_1 N_2 S_1 \mu}{l_1}$$

The value of μ is subject to the restrictions given for equation (570).

PROBLEMS.

150. Calculate the inductance of a solenoid 90 cm. long, an average diameter of 5 cm., and 5,000 turns.

151. A coil of 55 turns is 28 cm. long and 8 cm. in diameter. Calculate its inductance (in henries); when the core is of wood; when it is of iron having a permeability 1,200 times that of air.

152. Compare the inductance of a coil 20 cm. long, 8 cm. in diameter and of 50 turns about a core of permeability 1,200 times that of air, with an inductance of 2,000 turns on a wooden core 6 cm. diameter and 30 cm. long.

CHAPTER XVI.

ALTERNATING CURRENTS.

Circuits Containing Resistance, Inductance and Capacity.—If an elementary quantity of electricity, dq, is conveyed about an electric circuit back to its initial position, the final potential is the same as the initial and the work done *on* the charge is equal to the work done *by* the charge. This is the law of the conservation of energy applied to an electric circuit.

$$dw_1 = dw_2 + dw_3 + dw_4 + \cdots \tag{575}$$

Dividing by dq:

$$e_1 = e_2 + e_3 + e_2 + \cdots \quad \text{From (309).} \tag{576}$$

In which e_1 = work done per unit charge, in converting some other form of energy into electrical energy. It is called the *impressed e.m.f.*

e_2, e_3, e_4, etc., represent the work done per unit charge, in converting the electrical energy into some other form, such as heat, due to resistance, magnetic field due to the inductance, electrostatic field due to capacity, chemical energy due to electrolytic reaction, mechanical energy due to the force action on a conductor in a magnetic field, etc. These are called the "back e.m.f.'s." If in an electric circuit, containing only resistance, inductance, and capacity, e_2 be the back e.m.f. due to resistance,

(577) $$e_2 = R_1 i_1$$ (311)

If e_3 is the back e.m.f. due to self-induction,

(578) $$e_3 = L_1 \frac{di_1}{dt}$$ (462)

If e_4 = back e.m.f. due to the mutual inductance of a second circuit,

(579) $$e_4 = M \frac{di_2}{dt}$$ (469)

If e_4 = the back e.m.f. due to the charging of a condenser,

(580) $$e_5 = \frac{q_1}{C_1}$$ (315)

For a varying current,

(581) $$q_1 = \int i_1 dt$$ From (300).

(582) $$e_5 = \frac{1}{C_1} \int i_1 dt$$

Substituting these values for the back e.m.f.'s in (576),

(583) $$e_1 = R_1 i_1 + L_1 \frac{di_1}{dt} + M \frac{di_2}{dt} + \frac{1}{C} \int i_1 dt + \cdots$$

If the circuit contains only resistance and self-inductance,

(584) $$\therefore \quad e = Ri + L \frac{di}{dt}$$

Let

(585) $$i = I \sin \omega t$$

In which I = maximum value of i.

(586) $$\frac{di}{dt} = \omega I \cos \omega t$$

(587) $$\therefore \quad e = I(R \sin \omega t + L\omega \cos \omega t)$$

Multiply and divide by $\sqrt{R^2 + L^2\omega^2}$.

(588) $$e = I\sqrt{R^2 + L^2\omega^2}\left(\frac{R}{\sqrt{R^2 + L^2\omega^2}} \sin \omega t + \frac{L\omega}{\sqrt{(R^2 + L^2\omega^2}} \cos \omega t\right)$$

Let

(589) $$\cos \theta = \frac{R}{\sqrt{R^2 + L^2\omega^2}}$$

and let

(590) $$\sin \theta = \frac{L\omega}{\sqrt{R^2 + L^2\omega^2}}$$

(591) $$\therefore \quad \theta = \tan^{-1} \frac{L\omega}{R}$$

(592) $$e = I\sqrt{R^2 + L^2\omega^2} \cdot (\cos \theta \cdot \sin \omega t + \sin \theta \cos \omega t)$$

(593) $$\therefore \quad e = I\sqrt{R^2 + L^2\omega^2} \cdot \sin (\omega t + \theta)$$

The largest value of the last factor is 1.

$$\sin (\omega t + \theta) = 1$$

(594) $$E = I\sqrt{R^2 + L^2\omega^2}$$

Compare with (555).

In which E is the maximum value of e_1.

(595) $$e = E \sin(\omega t + \theta)$$

∴ An e.m.f. which is a sine function produces a current which is a sine function, having the same frequency and differing only in amplitude and phase.

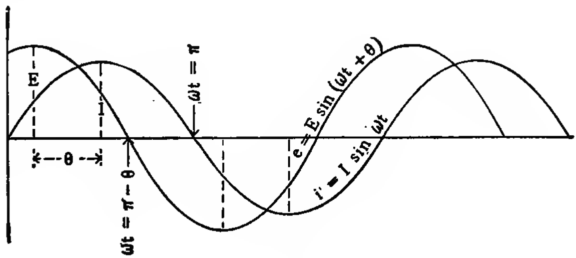


FIG. 81.

As the current and e.m.f. pass through a cycle of values, the current reaches its corresponding phases later in time and is said to "lag" behind the e.m.f. ∴ θ is called the "angle of lag" or phase difference.

Impedance.—Multiply both members of equation (594) by $\frac{1}{2}\sqrt{2}$ and combine with equation (569)

(596) $$E_{eff} = I_{eff}\sqrt{R^2 + L^2\omega^2}$$

In which I_{eff} is the effective value of the current. The ratio of the effective value of the e.m.f. to the effective value of the current is called "impedance." It is represented by the symbol Z.

$$Z = \frac{E_{\text{eff}}}{I_{\text{eff}}} \tag{597}$$

For a pure sine function involving only resistance and self-induction,

$$Z^2 = \sqrt{R^2 + L^2\omega^2} \tag{598}$$

If the circuit is composed of a non-inductive resistance,

$$L \doteq 0$$

$$\theta \doteq 0$$

$$\therefore \quad Z \doteq R \qquad \text{From (598).}$$

$$\therefore \quad e = Ri \quad \text{From (585) and (587).} \tag{599}$$

$\therefore$ Ohm's law holds not only for unvarying currents but also for circuits in which the current is in phase with the e.m.f.

CHAPTER XVII.

DYNAMO ELECTRIC MACHINERY.

In Chapter XIV there was developed the expression for an e.m.f. induced in a coil rotating in a magnetic field. Such a coil converts mechanical energy into electrical energy. Such a converter is called a "dynamo." If the current is not rectified it is called an alternating current dynamo or generator (A. C. generator); if the current is rectified or commutated, the machine is called a direct current dynamo or generator (D. C. generator).

Direct Current Generators.—The direct current generator consists of three essential parts, viz., the field, the commutator, and the armature or coil in which the e.m.f. is induced. If the field is that due to a permanent magnet it is called a "magneto." The field may be due to an electromagnet. If the magnetizing current is obtained from the generator itself, it is said to be a "self-exciting" generator. If the magnetizing current is obtained from some other source, the machine is said to be "separately excited." If the magnetizing coil is in series with the armature and the external circuit or line, a self-exciting machine is said to be "series wound." See Fig. 82. In this type of machine the current in the armature, field and line is the same. If, in a self-exciting machine, the magnetizing coil and the line are in parallel with each other and in series with the armature, the machine is said to be "shunt wound." See Fig. 83. The current in the armature is equal to the sum of the currents in the field and the line. If the magnetizing coil is composed of two parts, one of which is a series winding, the other a shunt winding, it is said to be "compound wound." See Fig. 84.

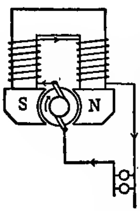


FIG. 82.

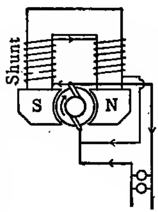


FIG. 83.

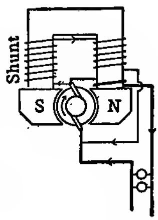


FIG. 84.

The curve showing variation of the voltage delivered to the line with the current in the line, or the load, is called "characteristic curve" of the machine. It differs for the three types of machines. See Fig. 85. For the same variation of load on the line, the compound wound machine gives most constant voltage. In the series machine, a certain value of the current in the line gives a maximum value of the voltage at the brushes. Currents greater or less than this give a lower value of the voltage. In the shunt wound machine an increase of the current in the line produces a lower voltage at the brushes. In the compound wound machine the voltage is more nearly a constant.

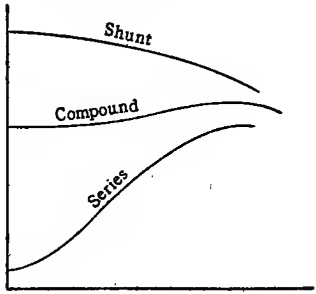


FIG. 85.

A change in the number of turns in the series coil will produce a marked change in the curve of this machine.

Armatures.—The value of the average e.m.f. induced in a coil rotating in a magnetic field is:

$$E_{av} = 4nA\beta f \tag{560}$$

or

(650) $$E_{av} = 4n\varphi f$$

In which φ = the maximum value of the flux enclosed by the coil, or the flux per pole.

If E_{av} is to be expressed in volts:

(651) $$E_{av} = \frac{4n\varphi f}{10^8}$$

In order to develop an e.m.f. of ordinary commercial value, it is necessary that φ should be fairly large. $\therefore$ The reluctance of the magnetic circuit should be fairly small.

$$\varphi = \frac{4\pi ni}{\dfrac{l_1}{\mu_1 s} + \dfrac{l_2}{s}} \tag{495}$$

This is accomplished by the use of an iron core in the coil, thus making the air-gap small.

Armature Cores.—Armature cores are of three types, shuttle or *H* or dumb-bell type, ring armatures, and drum armatures. Cross-sections of these are shown in Fig. 86. In general the windings on the two

H or shuttle type. Gramme or ring type. Siemens or drum type.

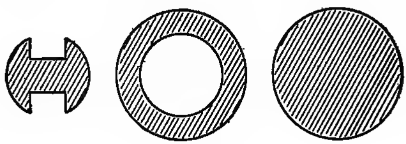


FIG. 86.

last types may be uniformly distributed or arranged in groups placed in slots which are cut in the outer surface of the core and parallel to its axis.

Open Coils.—If instead of half rings, Fig. 72, the commutator consists of quarter rings, the external circuit or "line" is in contact with the rotating coil for alternate quarter turns. See Fig. 87. The e.m.f. in the line is represented by the crests of the rectified

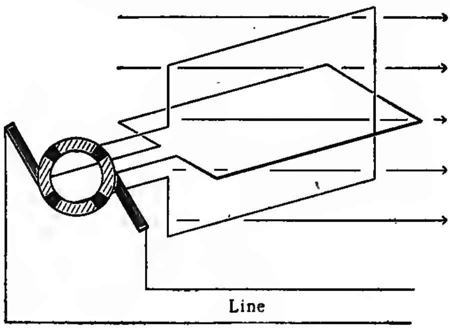


FIG. 87.

sine function, shown as "*a*," Fig. 88. If a second coil like this is mounted at right angles to the first and connected to quarter rings at right angles to the first two, the rectified e.m.f. in the line, due to this coil, will be represented by the crests shown as "*b*,"

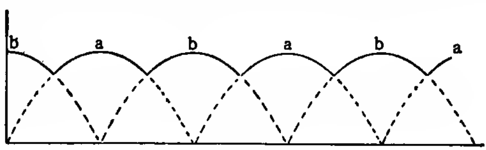


FIG. 88.

Fig. 88. By multiplying the number of coils and the number of commutator segments, the e.m.f. in the line may be made to approach a constant. Such a winding is called an "open coil winding."

Closed Coils.—The drum armature and the ring armature are usually wound as closed coils. See Fig. 89. If the number of commutator segments is large, the induced e.m.f. approaches a constant. Its value is the sum of the instantaneous values of the e.m.f. induced in the separate coils, or the number of coils times the average of the separate e.m.f. The average of the separate electromotive forces at any instant, is the average value of the electromotive force of a

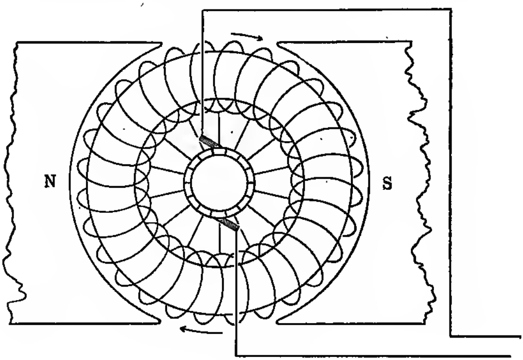


Fig. 89.

single coil for a half cycle. Therefore the total induced e.m.f. is equal to the total number of turns times the average value of the e.m.f. of a single turn.

$$\therefore \quad E_{av} = 4nA\beta f \qquad (560)$$

Characteristic Curves.—The action of the various parts of the generator may be most conveniently studied by means of curves showing the relation of the current and e.m.f. for these parts. Such curves, known as characteristic curves, may be obtained for particular machines by experimental or theoretical methods.

The self-induction of these generators is small.

$$\therefore \quad L \doteq 0$$

(652) $$\therefore E = E_1 + E_2 \qquad (576)$$

In which E = induced e.m.f.

E_1 = difference of potential across the brushes.

$E_2 = R_a I_a$ In the armature.

$E = 4nA\beta f$ (560)

(653) $$\therefore \quad 4nA\beta f = R_a I_a + E_1$$

The induction, however, is directly proportional to the current in the coils about the field and also proportional to the permeability.

$$\therefore \quad \beta = K\mu_f I_f$$

if the frequency is constant.

(654) $$\therefore \quad E_1 = K\mu_f I_f - R_a I_a$$

In which I_f = current in the field coil.

I_a = current in the armature.

K = some constant depending upon the number of turns in the field coil, the cross-section of the iron in the field, etc.

μ_f = permeability of iron in the field.

R_a = resistance of the armature.

Series Machine.—In the series wound generator the current in the armature, in the field and in the line is the same.

(655) $$I_f = I_a = I_l$$

If the frequency of this machine is kept constant and the load is varied, there will be a change of difference of potential across the brushes. Three curves are shown in Fig. 90; the upper one, which shows the

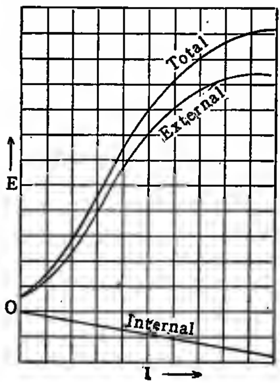


FIG. 90

relation between the total e.m.f. induced in the armature and the current in the field; the lower one, showing the relation between the fall of potential due to resistance of the armature and the current in the armature; the third, showing the relation between the current in the line and difference of potential at the brushes. The last curve may be obtained as the resultant of the first and second. Select any value of the current in the line. Draw the ordinate for this value. The total E generated, minus the E lost in the armature will equal the E at the brushes.

Shunt Machine.—If the speed of a shunt machine (Fig. 83) be kept constant and the load varied, there will be a change in the difference of potential at the brushes. In this case the current in the field does not increase so rapidly as the current in the line and the current in the armature coil increases more rapidly than the current in the line, as the resistance of the line is decreased.

(656) $$I_a = I_f + I_l$$

Curve A, Fig. 91, shows the relation of total e.m.f. developed, as ordinates, and current in the field coil, as abscissæ: The relation of the current in the line and in the field is given by the expression:

(657) $$I_f = \frac{R_l}{R_f} I_l$$

Curve B, Fig. 91, shows the relation of total e.m.f. developed, as ordinates, and the current in the line, as abscissæ. Curve *C*, Fig. 91, shows the relation of the fall of potential in the armature, as ordinates, and the

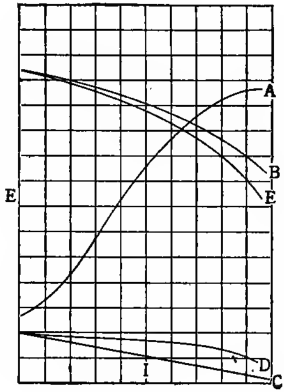


FIG. 91.

current in the armature as abscissæ. Substituting (657) in (656) there will be obtained:

$$I_a = \frac{R_f + R_l}{R_f} \cdot I_l \tag{658}$$

This relation makes it possible to plot the curve, *D*, Fig. 91, in which the fall of potential in the armature is taken as ordinate and the current in the line as abscissa. Curve *E*, the external characteristic, is the sum of *B* and *D*. It shows the relation of the fall of potential across the brushes as ordinates, and the current in the line as abscissæ.

Compound Wound Machine.—If a compound wound generator is rotated at a constant speed and the load altered, the change in e.m.f. at the brushes is not so marked as in case of the series or shunt wound machines. See Figs. 82, 83 and 84. The external characteristic is determined by the external characteristic of a shunt coil and of a series coil. See Fig. 92. Since the component due to the shunt coil

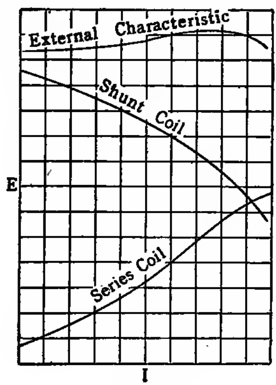


FIG. 92.

continues to decrease and the component due to the series coil continues to increase until it has reached a maximum, the resultant curve is nearly constant within the definite range of load for which the machine was designed.

Motors.—D. C. generators may be used as D. C. motors. The e.m.f. impressed at the brushes is opposed by the e.m.f. induced in the armature, due to its rotation, and the e.m.f. due to current and resistance in the armature or the RI drop. Let $e =$ the e.m.f. impressed.

$$e = e_1 + e_2$$

$$e_1 = R_a I_a$$

$$e_2 = \frac{d\varphi}{dt}$$

$e_2 =$ induced e.m.f. due to rotation. It is called the "back e.m.f."

$$\frac{d\varphi}{dt} = 4nA\beta f$$

(659) $$e = R_a I_a + 4nA\beta f$$

Multiply by I_a

(660) $$eI_a = R_a I_a + 4nA\beta f I_a$$

The electrical energy applied to the armature is converted into the work done by the motor and into heat, by the resistance of the armature. The power developed by the motor is equal to the power applied to the armature minus the power lost due to heat.

(661) $$P_m = P_a + P_H$$

In which $P_a =$ power applied to the armature.

$P_H =$ power lost in heat.

$P_m =$ power developed in motion.

(662) $$P_a = eI_a$$

(663) $$P_H = R_a I_a$$

(664) $$P_m = 4nA\beta f I_a$$

(665) $$\therefore \quad I_a = \frac{e - 4nA\beta f}{R_a}$$

For any given value of the power developed, the value of $\beta \cdot f$ is a constant. If then β is reduced, the speed of the machine will increase until the product of $\beta \cdot f$ has reached its former value. The speed does not however increase indefinitely if the field circuit is broken and the value of the induction reduced. The induction does not reach zero, due to the residual magnetism. There is then a definite maximum value of the speed which the machine will reach.

Transformer.—The instantaneous value of the e.m.f. produced in a coil rotating in a magnetic field, has been shown to be:

$$e = \omega A\beta \cos \omega t \tag{553}$$

If the terminals of the rotating coil are connected to rings instead of commutator segments, the e.m.f. in the external circuit will be the same as in the coil. A

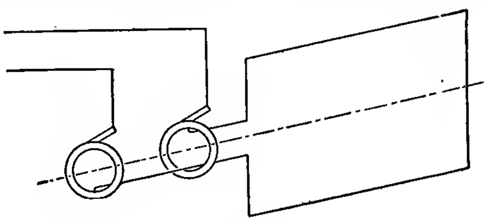


FIG. 93.

dynamo of this type is called a simple A. C. generator. Since the e.m.f. and the current in the line are alternating, current from this line cannot be used to excite the field coils of the machine itself. Separate D. C. generators are provided for this purpose or special coils are mounted on the same shaft and connected to a commutator.

The magnitude of the e.m.f. in an A. C. circuit can be changed by means of a device called a *transformer.* If the magnitude of the impressed e.m.f. is increased by the transformer, it is called a "step up" transformer; if decreased it is called a "step down" transformer. A typical transformer consists of two coils. a primary and a secondary, wound on the same iron core. See Fig. 94. If the entire magnetic circuit consists of iron it is called a closed core transformer. If the iron is in part of the circuit only, it is called an "open core" transformer. In good commercial forms of the "closed core" type, the flux per turn is approximately the same for every turn of the primary and of the secondary. The primary and secondary are typical magnetically related circuits.

If the e.m.f. from a simple A. C. generator is impressed upon one of the coils of the transformer and the other is on open circuit:

$$e_1 = R_1 i_1 + L_1 \cdot \frac{di_1}{dt} \tag{584}$$

$$L_1\frac{di_1}{dt} = \frac{d\varphi_1}{dt} \tag{461}$$

$$\varphi_1 = N_1\varphi$$

(666) $$e_1 = R_1i_1 + N_1\frac{d\varphi_1}{dt}$$

In which φ = flux per turn.

N_1 = number of turns in the primary.
φ_1 = flux through all turns of the primary.

Since in the transformer the coefficient of self-induction is large compared with the resistance,

$$e_1 \doteq N_1\frac{d\varphi_1}{dt}$$

or

$$\varphi = \frac{1}{N_1}\int e\cdot dt$$

But

$$e_1 = E_1 \cos \omega t \tag{595}$$

$$\therefore \quad \varphi = \frac{E_1}{N_1}\int \cos \omega t\cdot dt$$

$$\therefore \quad \varphi = \frac{-E_1}{N_1\omega}\cdot \sin \omega t$$

As the flux, φ due to the impressed e.m.f. in the primary varies, an e.m.f. is induced in the secondary:

$$e_2 = \frac{d\varphi_2}{dt}$$

$$\varphi_2 = N_2 \varphi$$

$$e_2 = N_2 \frac{d\varphi}{dt}$$

$$\varphi = \frac{-E_1}{N_1 \omega} \cdot \sin \omega t$$

$$\frac{d\varphi}{dt} = \frac{-E_1}{N_1} \cos \omega t$$

$$\therefore \quad e_2 = \frac{-N_2}{N_1} E \cos \omega t$$

(667) $$e_2 = \frac{-N_2}{N_1} e_1$$

The electromotive forces are directly proportional to the number of turns, provided the flux per turn is the same for all turns.

For an efficient transformer, *i. e.*, one in which the loss of power due to heat is small:

$$P_2 \doteq P_1$$

$$\therefore \quad e_1 i_1 = e_2 i_2$$

$$e_1 i_1 = -\frac{N_2}{N_1} e_1 i_2$$

(668) $$\therefore \quad i_1 = -\frac{N_2}{N_1} i_2$$

i. e., the electromotive forces are proportional to the number of turns and the currents are inversely as the number of turns.

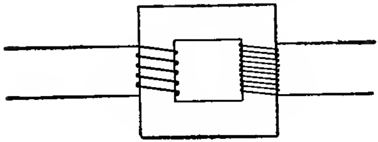


FIG. 94.

Induction Coil.—An induction coil is one type of open coil transformer. Its characteristic feature is: some device for automatically "making" and "breaking" a direct current in the primary. A simple form of such a device is shown by diagram in Fig. 95. In-

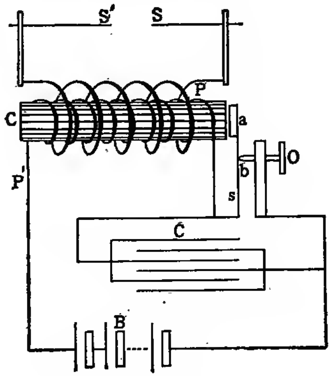


FIG. 95.

duction coils are used as step up transformers. A very high difference of potential can be produced by means of a very large number of turns in the secondary.

APPENDIX.

NATURAL FUNCTIONS OF SMALL ANGLES.

θ Degrees.	θ Radians.	sin θ.	tan θ.	cos θ.
1	.0174533	.01745	.01745	.99985
2	.0349066	.03490	.03492	.99939
3	.0523599	.05234	.05241	.99863
4	.0698132	.06976	.06993	.99756
5	.0872665	.08716	.08749	.99619
6	.1047198	.10453	.10510	.99452
7	.1221730	.12187	.12278	.99255
8	.1396263	.13917	.14054	.99027
9	.1570796	.15643	.15838	.98769
10	.1745329	.17365	.17633	.98481

Solid Angles.

If one point in a straight line remains fixed as the line is moved through any complete cycle back to its initial position, it generates a solid angle. The fixed point is called the apex of the angle. If a sphere is circumscribed with the apex as the center, the intersection of the surface of the solid angle and the sphere forms a closed curve. This curve divides the surface of the sphere into two parts. The smaller area is the measure of the acute solid angle. The area cut out on the sphere by the solid angle is proportional to the square of the radius. The unit of solid angle is a solid angle of such magnitude that the area cut out on a sphere, which has its center at the apex, is equal to the square of the radius. Since the area of a sphere is equal to $4\pi r^2$, the sum of all the solid angles about a point is equal to 4π units of solid angle.

Method of the Telescope and Scale.

Small angular deflections, like those in the magnetometer, tangent galvanometer or D'Arsonval galvanometer, are conveniently measured by means of a telescope and scale. A graduated scale is mounted on the same stand or fixture as the telescope which is provided with cross-hairs in the focus of the eye-piece. Rigidly attached to the body which makes the small angular deflection, there is mounted a small mirror. The telescope is pointed at the mirror and so adjusted that the reflected image of the scale is seen superposed upon the cross-hairs. When the mirror is turned through a small angle the image of the scale moves. The normal to the mirror moves through the same angle as the mirror but the apparent angular deflection is twice as great since the angle of incidence is equal to the angle of reflection. The angular deflection is determined from the distance between telescope and mirror, and the deflection of the scale as observed in the telescope.

$$\tan 2\theta = \frac{S}{d}$$

International Units.

The International Conference on Electrical Units and Standards has adopted the following resolutions:

I. The Conference agrees that as heretofore the magnitudes of the fundamental electric units shall be determined on the electro-magnetic system of measurement with reference to the centimeter as the unit of length, the gram as the unit of mass, and the second as the unit of time. . . .

II. As a system of units representing the above and sufficiently near to them to be adopted for the purpose of electrical measurements and as a basis for legislation, the Conference recommends the adoption of the International Ohm, the International Ampere, and the International Volt. . . .

V. The International Ohm is the resistance offered to an unvarying electric current by a column of mercury at the temperature of melting ice, 14.4521 grammes in mass, of a constant cross-sectional area and of a length of 106.300 centimeters. . . .

VII. The International Ampere is the unvarying electric current which, when passed through a solution of nitrate of silver in water (in accordance with specifications attached to the resolutions*) deposits silver at the rate of 0.00111800 of a gramme per second.

* See Report of the International Conference on Electrical Units and Standards, 1908. Wyman & Son, Ltd., Fetter Lane, E. C. London.

VIII. The International Volt is the electrical pressure which, when steadily applied to a conductor whose resistance is one International Ohm, will produce a current of one International Ampere.

IX. The International Watt is the energy expended per second by an unvarying electric current of one International Ampere under an electric pressure of one International Volt.

UNITS.

Quantity.	Symbol.	Dimension of Unit.	Name of Practical Unit.	Equivalent Number of c.g.s. Units
(Electrostatic)				
Specific inductive capacity	κ	$[\kappa]$		
Quantity of charge	q	$[\kappa^{1/2}M^{1/2}L^{3/2}T^{-1}]$		
Intensity of field	$\mathcal{F}$	$[\kappa^{-1/2}M^{1/2}L^{-1/2}T^{-1}]$		
Induction (electrostatic)	$\mathfrak{N}$	$[\kappa^{1/2}M^{1/2}L^{-1/2}T^{-1}]$		
Normal induction	N	$[\kappa^{1/2}M^{1/2}L^{3/2}T^{-1}]$		
Difference of potential	e	$[\kappa^{1/2}M^{1/2}L^{1/2}T^{-1}]$		
Capacity	C	$[\kappa L]$		
(Magnetic)				
Permeability	μ	$[\mu]$		
Quantity of magnetism	m	$[\mu^{1/2}M^{1/2}L^{3/2}T^{-1}]$		
Intensity of field	$\mathcal{H}$	$[\mu^{-1/2}M^{1/2}L^{-1/2}T^{-1}]$		
Induction (magnetic)	β	$[\mu^{1/2}M^{1/2}L^{-1/2}T^{-1}]$		
Flux	φ	$[\mu^{1/2}M^{1/2}L^{3/2}T^{-1}]$	weber	10^8
(Electromagnetic)				
Current	i	$[\mu^{-1/2}M^{1/2}L^{1/2}T^{-1}]$	ampere	10^{-1}
Quantity	q	$[\mu^{1/2}M^{1/2}L^{1/2}]$	coulomb	10^{-1}
Resistance	R	$[\mu LT^{-1}]$	ohm	10^9
Difference of potential	e	$[\mu^{1/2}M^{1/2}L^{3/2}T^{-2}]$	volt	10^8
Capacity	C	$[\mu^{-1}L^{-1}T^2]$	farad	10^{-9}
			micro-farad	10^{-15}
Inductance	L	$[\mu L]$	henry	10^9
			milli-henry	10^6

Important Formulæ.

(1) $$f = \frac{1}{\mu} \cdot \frac{m \cdot m'}{r^2}$$

(2) $$f = m' \cdot \mathcal{H}$$

(9) $$\mathcal{H} = \frac{1}{\mu} \cdot \sum_{k=1}^{n} \frac{m_k}{{r_k}^2} \cos \alpha_k$$

(12) $$\mathcal{H}_r = \sqrt{\mathcal{H}_x^2 - \mathcal{H}_y^2 + \mathcal{H}_z^2}$$

(18) $$\tan \alpha = \frac{\mathcal{H}_y}{\mathcal{H}_x}$$

(21) $$M = m \cdot l$$

(22) $$L = M \cdot \mathcal{H} \cdot \sin \theta$$

(25) $$\tan \theta_e = \frac{\mathcal{H}_m}{\mathcal{H}_e}$$

(26) $$\mathcal{H}_a = \frac{2 \cdot M \cdot d}{\left(d^2 - \frac{l^2}{4}\right)^2}$$

(28) $$\mathcal{H}_a = \frac{2 \cdot M}{d^3}$$

(32) $$\mathcal{H}_p = \frac{M}{\left(d^2 + \frac{l^2}{4}\right)^{3/2}}$$

(34) $$\mathcal{H}_p = \frac{M}{d^3}$$

(45) $$T = 2 \cdot \pi \sqrt{\frac{I}{M \mathcal{H}_e}}$$

(50) $$\beta = \mu \cdot \mathcal{H}$$

(51) $$\beta = \frac{m}{r^2}$$

(56) $$\varphi = S\beta \cos \alpha$$

(58) $$\Phi = \Sigma\beta S \cos \alpha$$

(60) $$\Phi = 4\pi m$$

(67) $$\Phi = \beta \cdot S$$

(71) $$\beta_2 \cos \alpha = \beta_1 \cos \alpha + 4\pi\mathcal{J}$$

(72) $$\beta_2 = \beta_1 + 4\pi\mathcal{J}$$

(100) $$f = \frac{1}{\kappa}\frac{q \cdot q'}{r^2}$$

(101) $$f = \mathcal{F}q'$$

(108) $$\mathcal{F}_x = \frac{1}{\kappa}\sum_{k=1}^{n}\frac{q_k}{r_k{}^2}\cos \alpha_k$$

(111) $$\mathcal{F}_r = \sqrt{\mathcal{F}_x{}^2 + \mathcal{F}_y{}^2 + \mathcal{F}_z{}^2}$$

(117) $$\tan \alpha = \frac{\mathcal{F}_y}{\mathcal{F}_x}$$

(118) $$e = \frac{w}{q'}$$

(126) $$e = \frac{q}{\kappa}\left(\frac{1}{r_0} - \frac{1}{r_n}\right)$$

(127) $$V = \frac{q}{\kappa} \cdot \frac{1}{r_0}$$

(129) $$e = V_1 - V_2$$

(130) $$\mathfrak{D} = \kappa\mathcal{F}$$

(131) $$\mathcal{H} = \frac{q}{r^2}$$

(134) $$\mathrm{N} = \mathcal{H} \cdot S \cdot \cos\alpha$$

(136) $$N = \Sigma \mathcal{H} \cdot S \cdot \cos\alpha$$

(137) $$N = 4 \cdot \pi \cdot q$$

(143) $$N = \mathcal{H} \cdot S$$

(146) $$\mathcal{F} = \frac{1}{\kappa} \cdot \frac{2\rho}{r}$$

(148) $$\mathcal{F} = \frac{4\pi\sigma}{\kappa}$$

(155) $$\mathcal{F} = \frac{2\pi\sigma}{\kappa}$$

(157) $$\mathcal{F} = \frac{4\pi\sigma}{\kappa}$$

(160) $$\mathcal{F} = \frac{1}{\kappa}\frac{q}{r^2}$$

(164) $$e = \sqrt{\frac{8\pi d^2}{A} \cdot f}$$

(200) $$C = \frac{q}{e}$$

(201) $$C = \frac{q}{V}$$

(202) $$C = \kappa r$$

(204) $$C = \kappa\left(\frac{r_1 r_2}{r_2 - r_1}\right)$$

16

(205) $$C = \kappa \frac{A}{4 \cdot \pi \cdot d}$$

(206) $$C = C_1 + C_2 + C_3 + \cdots$$

(208) $$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots$$

(210) $$C = \kappa \frac{(n-1)A}{4\pi d}$$

(214) $$\frac{C_1}{C_2} = \frac{e_2}{e_1 - e_2}$$

(216) $$\kappa = \frac{e_2}{e_1 - e_2}$$

(220) $$w = \tfrac{1}{2}eq$$

(223) $$\frac{\mathcal{F}_1}{\mathcal{F}_2} = \frac{r_2}{r_1}$$

(229) $$\mathcal{H}_x = \int_0^l \frac{i \cdot \sin\theta \cdot \cos\varphi_X \cdot dl}{r^2}$$

(237) $$i = \frac{a}{2 \cdot \pi \cdot n} \mathcal{H}_x$$

(238) $$i = \frac{a}{2\pi n} \cdot \mathcal{H}_\iota \cdot \tan\theta_i$$

(242) $$\mathcal{H}_x = \frac{2\pi n a^2}{(a^2 + x^2)^{\frac{3}{2}}} \cdot i$$

(248) $$\mathcal{H}_x = 4 \cdot \pi \cdot \frac{N}{l} \cdot i$$

(250) $$\mathcal{H}_x = \frac{2 \cdot i}{d}$$

(300) $$q = it$$

(301) $$m_2 - m_1 = cit$$

(304) $$q = 96530 \frac{m_2 - m_1}{\text{combining weight}}$$

(305) $$w = R \cdot i^2 \cdot t$$

(308) $$w = (M + c) \cdot \theta \cdot (4.2)(10)^7$$

(309) $$e = \frac{w}{q}$$

(311) $$e = Ri$$

(320) $$\Sigma i = 0$$

(321) $$\Sigma e = 0$$

(324) $$R = R_1 + R_2 + R_3 + \cdots$$

(328) $$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots$$

(332) $$R = \gamma \cdot \frac{l}{s}$$

(334) $$R = R_0(1 + \alpha T)$$

(342) $$\frac{R_1}{R_2} = \frac{R_3}{R_4}$$

(346) $$R_X = \frac{l_1}{l_2} R_s$$

(358) $$\frac{e_X}{e_S} = \frac{l_1}{l_2}$$

(361) $$p = ei$$

(362) $$p = Ri^2$$

(401) $$f_x = \int \beta_m \cdot i \cdot \sin\theta \cdot \cos\varphi_x \cdot dl$$

(404) $$f = l \cdot \beta \cdot i$$

(406) $$L = n \cdot A \cdot \beta \cdot i \cdot \cos\theta_r$$

(413) $$i = K \cdot D$$

(414) $$i = K\frac{R_c + R_s}{R_s}D$$

(418) $$e = K \cdot (R_m + R_c) \cdot D$$

(425) $$dw = e \cdot i \cdot dt$$

(450) $$e = \frac{d\varphi}{dt}$$

(455) $$e = l \cdot \beta \cdot v$$

(460) $$L = \frac{d\varphi_1}{di_1}$$

(462) $$e = L\frac{di_1}{dt}$$

(465) $$M = \frac{d\varphi_1}{di_2}$$

(469) $$e = M\frac{di_2}{dt}$$

(471) $$w = \int e \cdot i \cdot dt$$

(478) $$w = 4\pi N m i$$

(481) $$\mathcal{H} = \frac{4\pi N i}{l}$$

(486) $$\varphi = \frac{4\pi N i}{\left(\frac{l}{\mu S}\right)}$$

(492) $$\varphi = \frac{\mathcal{M}}{\mathcal{R}}$$

(499) $$\mu_1 \mathcal{H} = \mu_2 \mathcal{H} + 4\pi \mathcal{J}$$

(500) $$\beta = \mathcal{H} + 4\pi \mathcal{J}$$

(554) $$e = n \cdot \omega \cdot A \cdot \beta \cdot \cos \omega t$$

(556) $$e = E_{\max} \cos \omega t$$

(561) $$E_{av} = \frac{2}{\pi} E_{\max}$$

(565) $$E_{eff} = \sqrt{\frac{8\pi d^2}{A} F_{av}}$$

(569) $$E_{eff} = E_{\max} \tfrac{1}{2} \sqrt{2}$$

(570) $$L = \frac{4\pi N_1^2 S_1 \mu}{l_1}$$

(571) $$M = \frac{4\pi N_1 N_2 S_1 \mu}{l_1}$$

(583) $$e_1 = R_1 i_1 + L_1 \frac{di_1}{dt} + M \frac{di_2}{dt} + \frac{1}{C} \int i_1 dt + \cdots$$

(588)
$$e = I\sqrt{R^2 + L^2\omega^2}\left(\frac{R}{\sqrt{R^2 + L^2\omega^2}} \cdot \sin \omega t + \frac{R}{\sqrt{R^2 + L^2\omega^2}} \cdot \cos \omega t\right)$$

(595)
$$e = E \cdot \sin(\omega t + \theta)$$

(598)
$$Z = \sqrt{R^2 + L^2\omega^2}$$

(654)
$$E_1 = K\mu_f I_f - R_a I_a$$

(660)
$$eI_a = R_a I_a + 4 \cdot n \cdot A \cdot \beta \cdot f \cdot I_a$$

(667)
$$e_2 = \frac{-N_2}{N_1} e_1$$

(668)
$$\dot{i}_1 = -\frac{N_2}{N_1} \dot{i}_2$$

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